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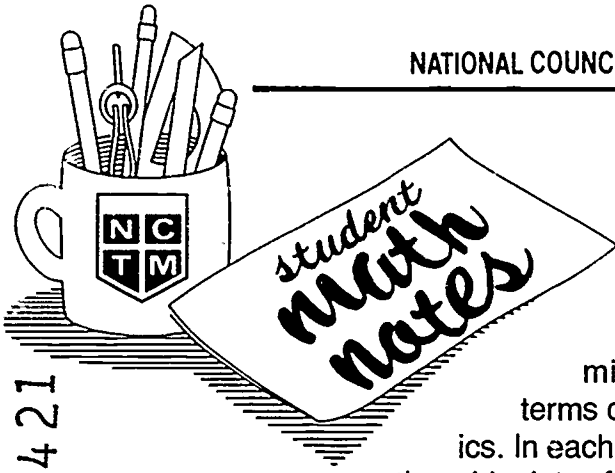
ABSTRACT

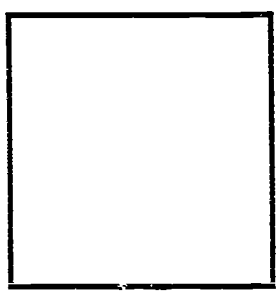
Five sets of activities for students are included in this document. Each is designed for use in junior high and secondary school mathematics instruction. The first two are concerned with the properties of polygons. The third involves statistical decision making using basketball statistics as examples. The fourth is related to the perimeter and area of squares and rectangles. The fifth describes the uses of directed graphs. Each set contains a variety of activities, an extension section, and an answer key. (CW)



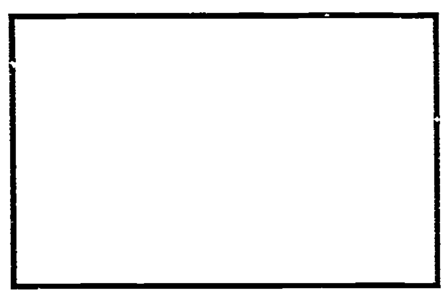
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS — JANUARY 1988

Midpoint Madness

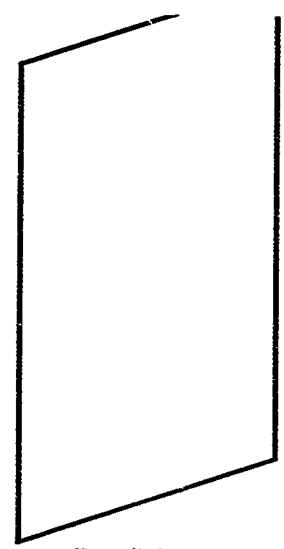
The concept of midpoint is very rich in terms of its usefulness in mathematics. In each of the following quadrilaterals find the midpoints of the sides and label them consecutively *N, C, T,* and *M*. Then draw the segments forming the polygon *NCTM*.



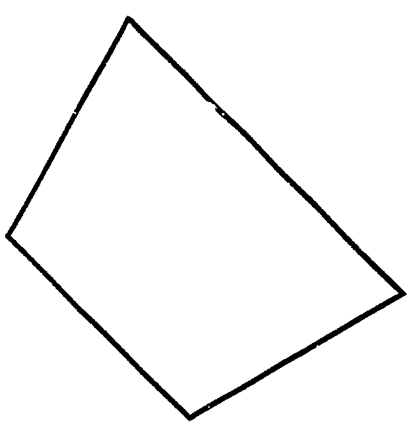
Square



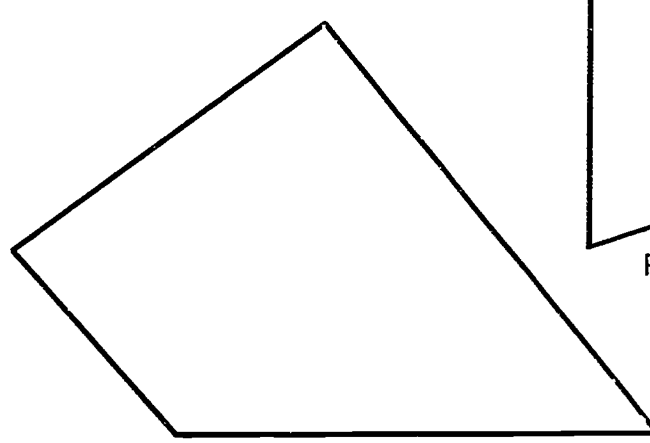
Rectangle



Parallelogram



Trapezoid



General quadrilateral

What appears to be true about all the *NCTM* polygons? ______________________

____

What also appears to be true about the relationship between the area of each *NCTM* polygon and the area of its original quadrilateral? ______________________

____

For centuries mathematics students have studied a theorem stating that connecting the midpoints of consecutive sides of any quadrilateral will always form a parallelogram and that the area of the parallelogram is one-half the area of the original quadrilateral.

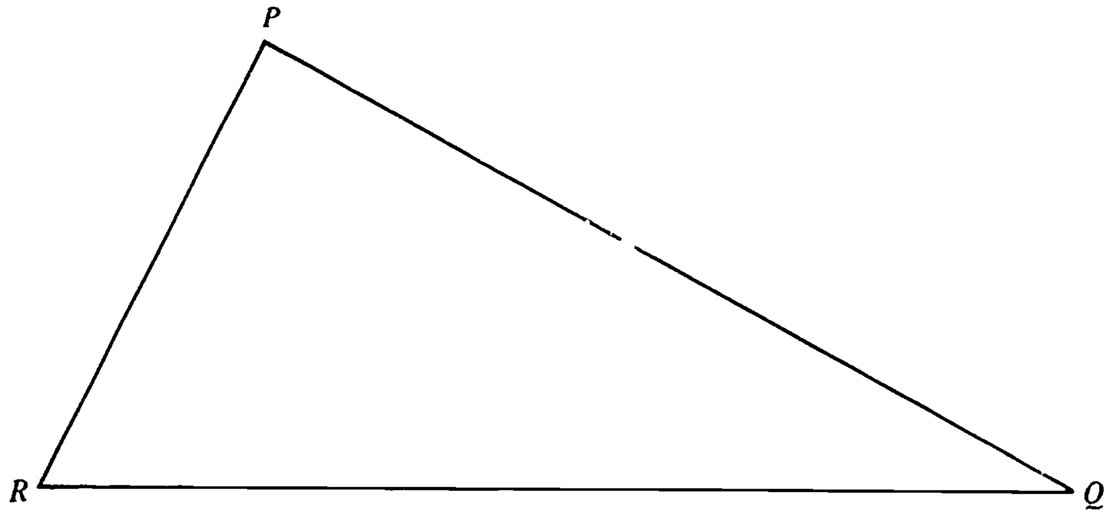


Fig. 1

In $\triangle PQR$, locate the midpoints of sides $\overline{PQ}$, $\overline{QR}$, and $\overline{RP}$ and label them A_1, B_1, and C_1, respectively. Connect these midpoints, forming $\triangle A_1B_1C_1$.

- How do $m\angle P$ and $m\angle A_1B_1C_1$ compare? ____________
- How do $m\angle Q$ and $m\angle A_1C_1B_1$ compare? ____________
- How do $\triangle PQR$ and $\triangle B_1C_1A_1$ compare in terms of shape? ____________
- How do they compare in terms of area? ____________
- What kinds of quadrilaterals are polygons $A_1B_1C_1P$, $A_1QB_1C_1$, and $A_1B_1RC_1$?

  ____

  Why? ____________
- What kinds of quadrilaterals are polygons A_1B_1RP, C_1A_1QR, and B_1C_1PQ?

  ____

  Why? ____________

Find the midpoints of the sides of $\triangle A_1B_1C_1$ and name them A_2, B_2, and C_2. (You may label them in any order.)

- How do $\triangle PQR$, $\triangle A_1B_1C_1$, and $\triangle A_2B_2C_2$ compare in terms of shapes?

  ____

  How do they compare in terms of areas? ____________

Repeat the process of successively finding midpoints of the sides of the newly formed interior triangles, labeling them A_i, B_i, and C_i as the number of triangles increases from 1 to 2 to 3 to i.

- What do you notice about the points A_i, B_i, and C_i as i increases?

  ____

Draw in the medians $\overline{RA_1}$, $\overline{PB_1}$, and $\overline{QC_1}$. Label their point of intersection O.

- What do you notice about O and the points A_i, B_i, and C_i as i increases?

  ____

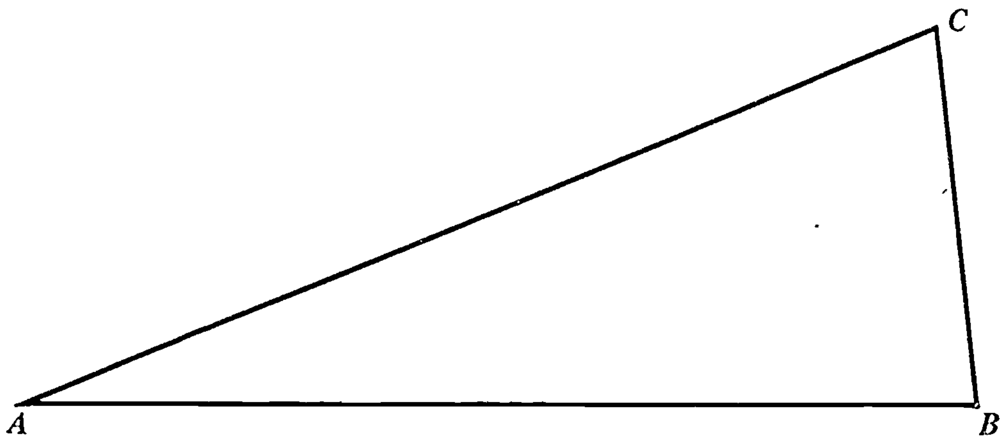


Fig. 2

In $\triangle ABC$, locate D_1, the midpoint of $\overline{AC}$, and E_1, the midpoint of $\overline{BC}$. Draw $\overline{D_1E_1}$; measure $\overline{D_1E_1}$ and $\overline{AB}$.

D_1E_1 = ____________; AB = ____________.

- How does D_1E_1 compare to AB? ______________________________

  Draw $\overline{D_1B}$ and $\overline{E_1A}$; label the poin. ᵓf intersection F_1.

Locate D_2, the midpoint of $\overline{AF_1}$, and E_2, the midpoint of $\overline{BF_1}$. Draw $\overline{D_2E_2}$; measure $\overline{D_2E_2}$.

D_2E_2 = ____________.

- How does D_2E_2 compare to D_1E_1? ______________________________
- How does D_2E_2 compare to AB? ______________________________

Draw $\overline{D_2B}$ and $\overline{E_2A}$; label the point of intersection F_2.

Locate D_3, the midpoint of $\overline{AF_2}$, and E_3, the midpoint of $\overline{BF_2}$. Draw $\overline{D_3E_3}$; measure $\overline{D_3E_3}$. D_3E_3 = ____________.

- How does D_3E_3 compare to D_1E_1 and D_2E_2?

  ____

- How does D_3E_3 compare to AB?

  ____

- If we continue, how would the measures of all the segments D_iE_i compare to one another?

  ____

- If we continue, how would the measures of all the segments D_iE_i compare to the measure of $\overline{AB}$?

  ____

- What do you notice about the points F_i as i increases?

  ____

Draw $\overline{D_1D_2}$, $\overline{D_2D_3}$, $\overline{E_1E_2}$, $\overline{E_2E_3}$, and so on.

- What kind of quadrilateral is $D_1D_2E_2E_1$ or $D_2D_3E_3E_2$ or $D_1D_3E_3E_1$? ______________________________

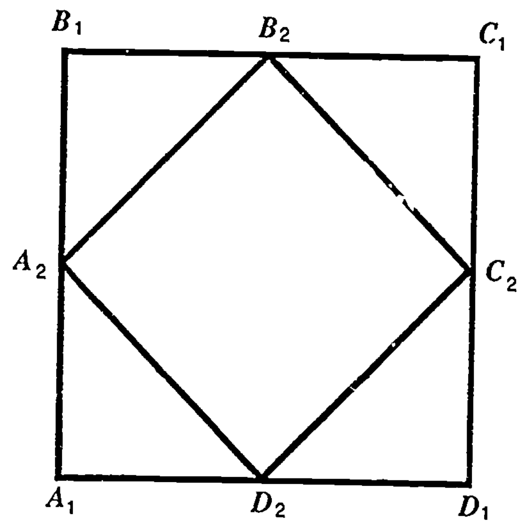
  Why? ______________________________

- What kind of quadrilateral is AD_1E_1B or AD_2E_2B or AD_3E_3B? ______________ ______________

  Why? ______________________________

4

The midpoints of each side were located in the unit square $A_1B_1C_1D_1$. The square $A_2B_2C_2D_2$ was then constructed.



How does the area of $A_2B_2C_2D_2$ compare to the area of $A_1B_1C_1D_1$? __________

Now locate the midpoints of each side of $A_2B_2C_2D_2$ and label them A_3, B_3, C_3, and D_3 to form the square $A_3B_3C_3D_3$.

How does the area of $A_3B_3C_3D_3$ compare to the area of $A_1B_1C_1D_1$? __________

Continue to generate several more squares using the process described previously. How does the area of $A_5B_5C_5D_5$ compare to the area of $A_1B_1C_1D_1$? __________

How does the area of $A_9B_9C_9D_9$ compare to the area of $A_1B_1C_1D_1$? __________

What is the area of $A_iB_iC_iD_i$ approaching as i gets larger? __________

With a colored pencil, connect segments A_1A_2, A_2A_3, A_3A_4, and A_4A_5.

1. What is the sum of the measures of these four segments? __________
2. What is the sum of the measures of n segments (as n gets extremely large)? __________
3. Assume that the squares are generated ad infinitum. What is the measure of $A_1A_2 + A_2A_3 + A_3A_4 + \ldots$?

Did you know that . . .

- the point O in figure 1 is called the centroid of triangle PQR? In physics, centroids are known as the center of mass.
- the sequence ½, ¼, ⅛, . . . , $\frac{1}{2^n}$ is known as a geometric sequence?
- when the terms of a geometric sequence are added together, a geometric series is formed?
- the sum of the geometric series 1 + ½ + ¼ + ⅛ + . . . is 2?

Can you . . .

- prove that the *NCTM* polygons are all parallelograms?
- prove that the area of each *NCTM* polygon is half the area of its original quadrilateral?
- prove that the area of $\triangle A_1B_1C_1$ in figure 1 is one-fourth the area of $\triangle PQR$?
- prove that the area of $\triangle CD_1E_1$ in figure 2 is one-fourth the area of $\triangle ABC$?

Solutions

1. $2(1-(\frac{1}{2})^n)$; 2. $\frac{10+5\sqrt{2}}{16}$; 3. $\frac{2+\sqrt{2}}{3}$

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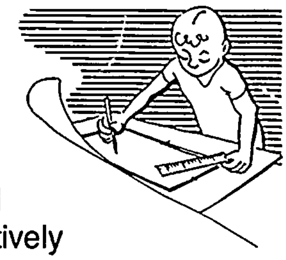
Midpoint Madness



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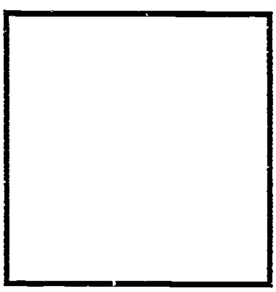
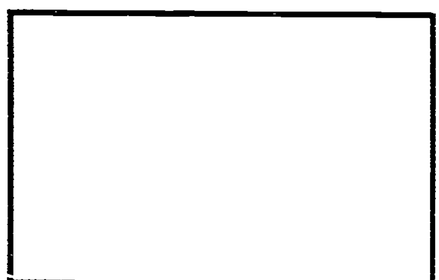
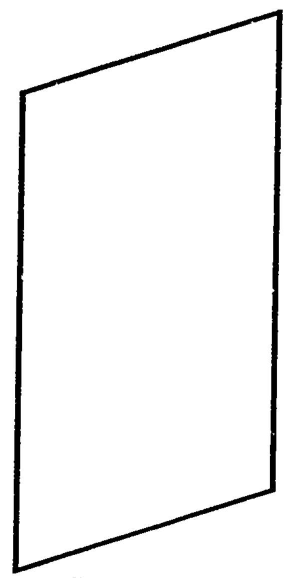


Square



Rectangle



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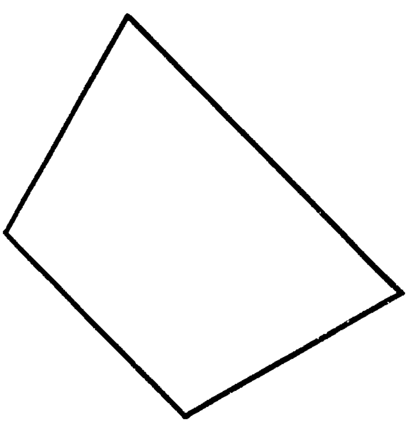

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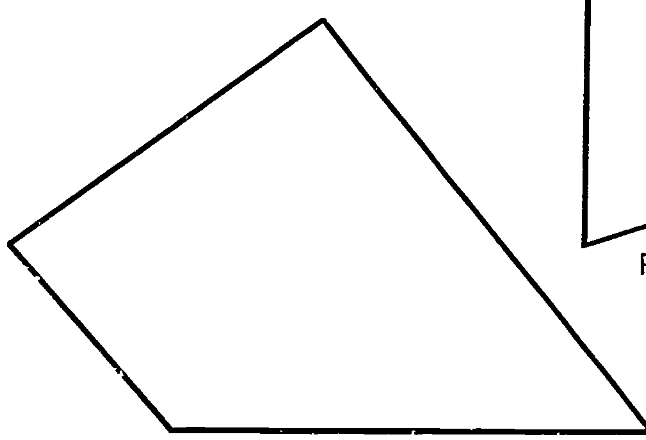

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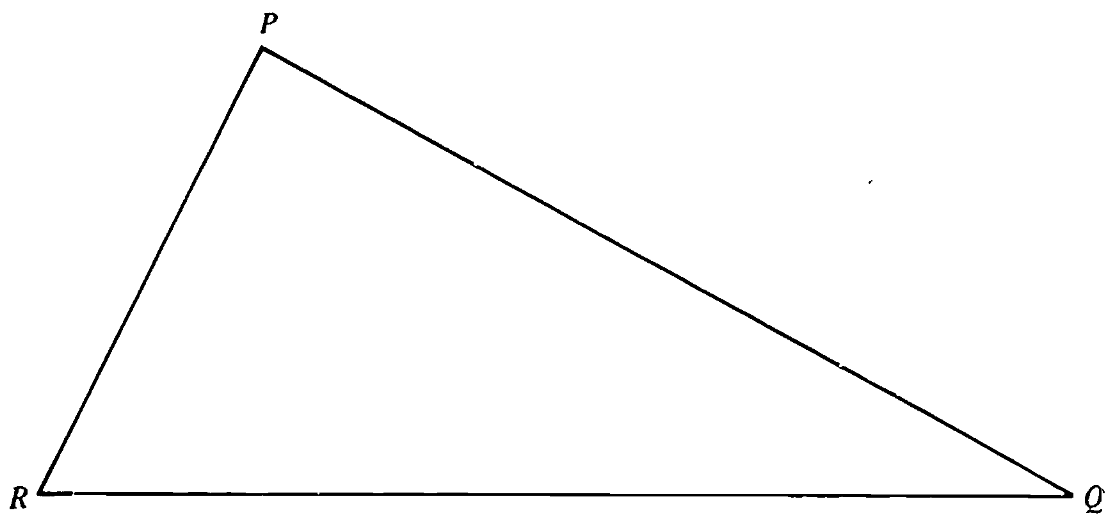

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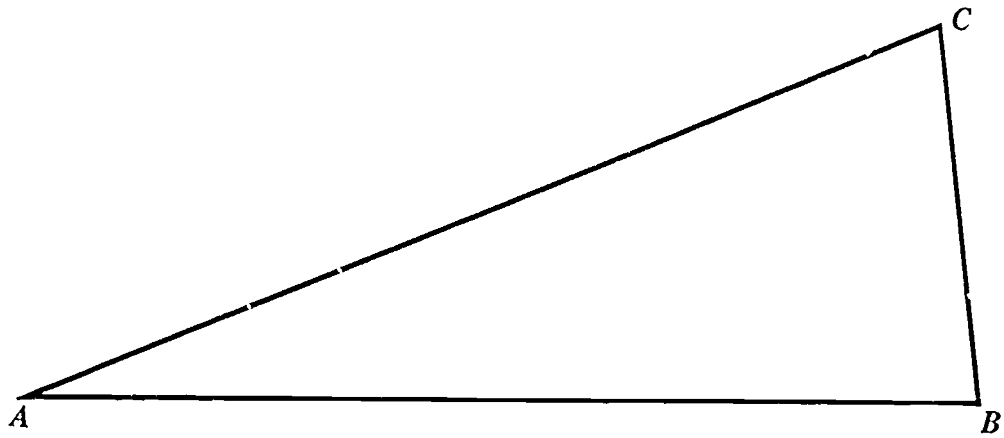

Fig. 2

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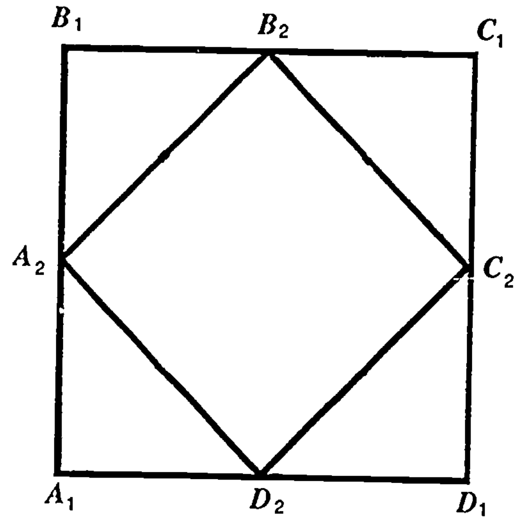

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____

How does the area of $A_9B_9C_9D_9$ compare to the area of $A_1B_1C_1D_1$? ______________________________

What is the area of $A_iB_iC_iD_i$ approaching as i gets larger? ______________________________

With a colored pencil, connect segments A_1A_2, A_2A_3, A_3A_4, and A_4A_5.

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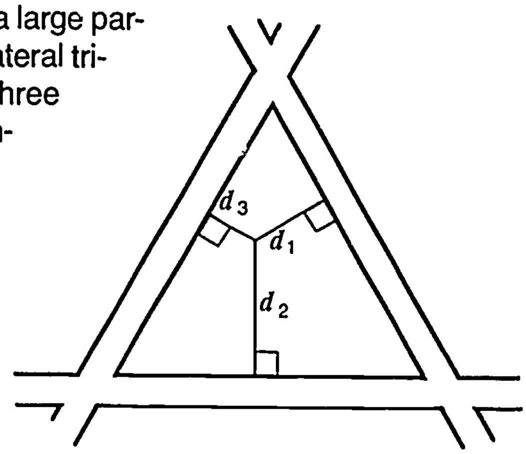
student math notes

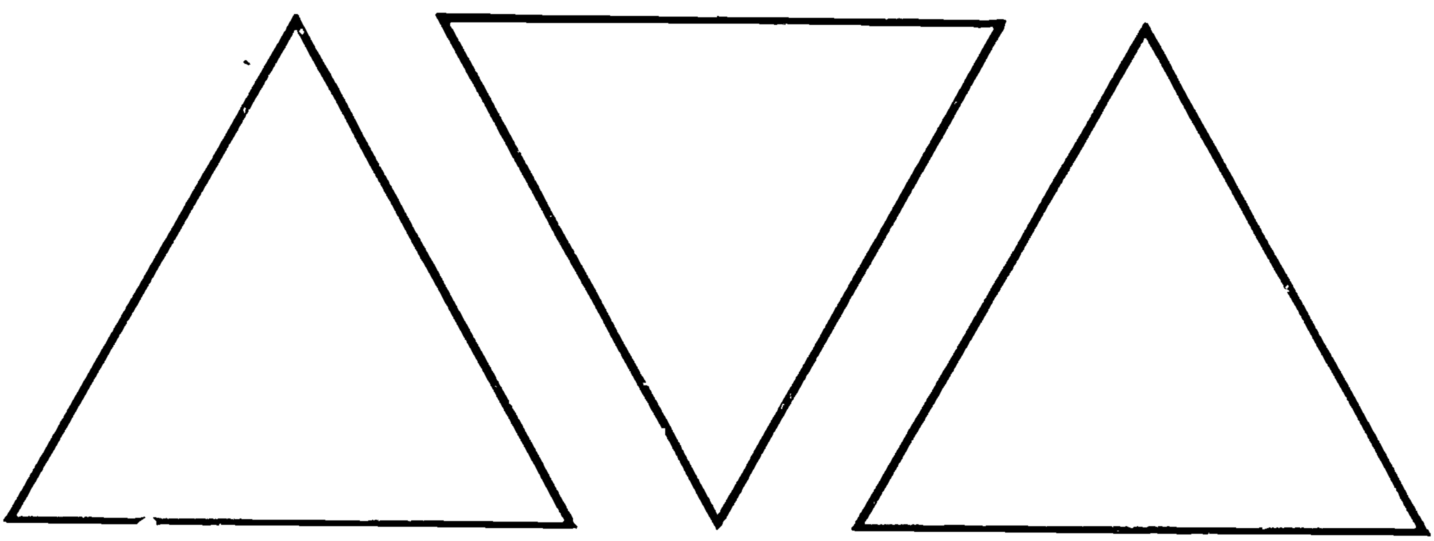
Minimum Distance

Stu Dent recently inherited a large parcel of land shaped like an equilateral triangle. The land's boundaries are three highways, intersecting to form the triangle's three vertices. However, for Stu to receive the land, the will required that he build a house on the triangular parcel of land and also that he build three roads from the house, each perpendicular to one of the three highways. Furthermore, the will required that the sum of the distances, $d_1 + d_2 + d_3$, from the house to the highways *must* be the **smallest** possible.
Where should Stu put his house?



Let's Explore Some Possibilities for Stu

Using a ruler and protractor, locate possible points—housing sites—on or within the triangles below and construct the three roads from the house perpendicular to the highways. Measure and record the **sum** of the three distances under each triangle.

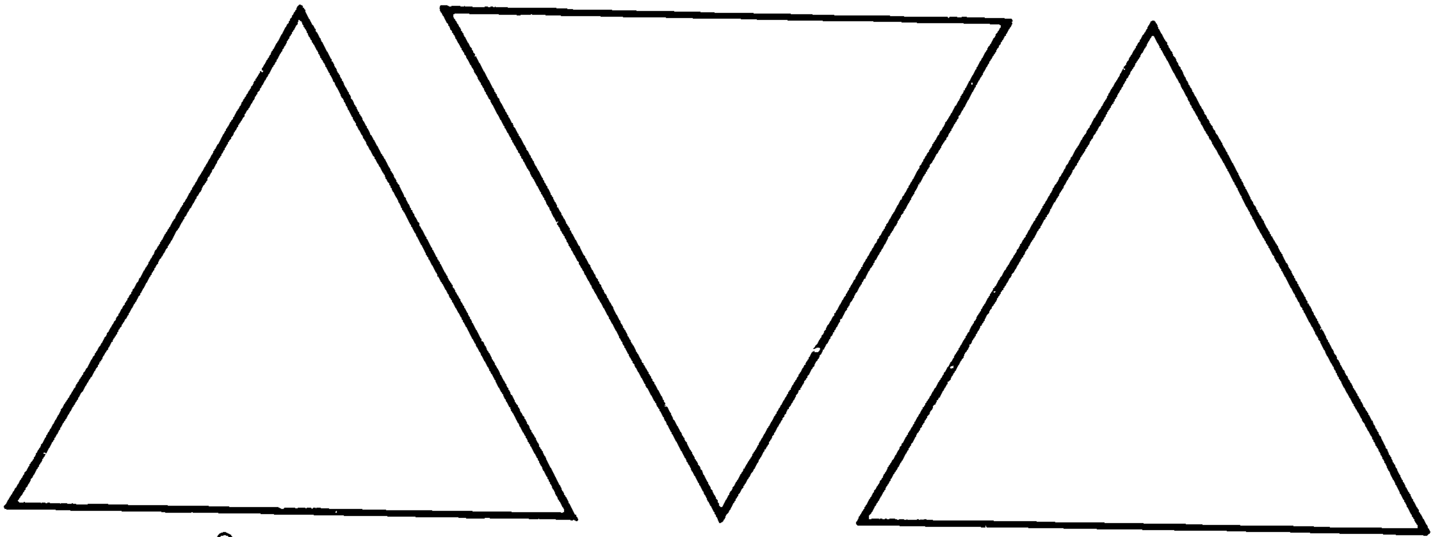


Sum = ___ Sum = ___ Sum = ___

The editors wish to thank Jerry L. Johnson, Western Washington University, Bellingham, WA 98225, for writing this issue of the *NCTM Student Math Notes*.

Continuing Your Explorations

Now try some additional points. Be sure to try special points, such as the intersection of the medians, a vertex, or a point on the edge of the parcel of land.



Sum = ___ Sum = ___ Sum = ___

- What is the sum of the distances whenever the point chosen is—

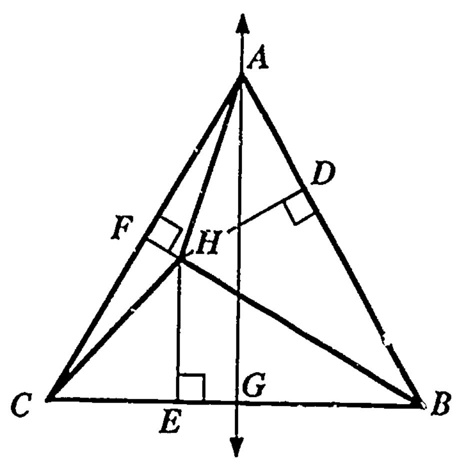
  within the triangle? ______________ on the edge of the triangle? ____________

  at a vertex of the triangle? ____________

- What appears to be true about all these sums? ________________________

Verifying Your Discovery

The diagram at the right represents the conditions of the will. Point H was selected at random to represent the location of the house with $\overline{HD}$, $\overline{HE}$, and $\overline{HF}$ representing the three roads. Notice that the area of $\triangle ABC$ has been divided into three smaller triangles by $\overline{HC}$, $\overline{HB}$, and $\overline{HA}$, and that



area of $\triangle ABC$ = area of $\triangle BCH$ + area of $\triangle ACH$ + area of $\triangle ABH$.

Using the area formula, $A = (1/2)bh$, we have

$$\frac{1}{2}(BC)(AG) = \frac{1}{2}(BC)(HE) + \frac{1}{2}(AC)(HF) + \frac{1}{2}(AB)(HD).$$

Since $\triangle ABC$ is equilateral, the sides have equal measure. Therefore, by substituting BC for AC and AB, we have

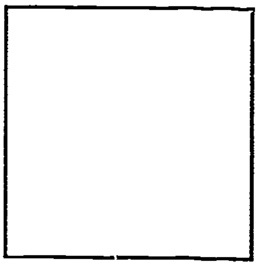
$$\frac{1}{2}(BC)(AG) = \frac{1}{2}(BC)(HE) + \frac{1}{2}(BC)(HF) + \frac{1}{2}(BC)(HD)$$
$$= \frac{1}{2}(BC)(HE + HF + HD).$$

By comparing both sides of this equation, we can see that $AG = HE + HF + HD$.

Therefore the sum of the three perpendicular segments, or roads, is a constant value. Regardless of where the house is located, the sum of the segments is equal to the length of the altitude of the original equilateral triangle, or land parcel. Now, construct an altitude in one of the triangles at the top of the page and measure its length to verify your discovery. Is its length equal to your sums? __________

Further Explorations

If the parcel of land in the will had been in the shape of a square, where should Stu locate the house for the sum of the distances to the four sides to be a minimum?



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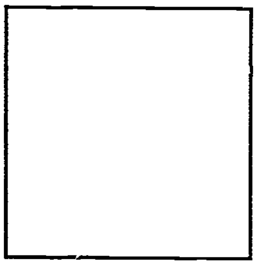
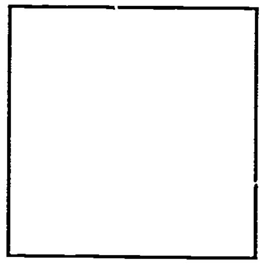


Sum = ___



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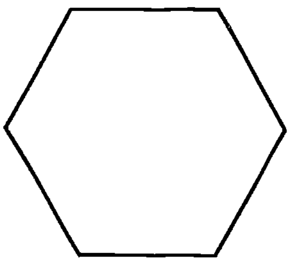

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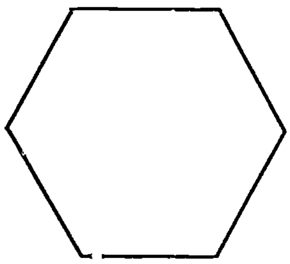

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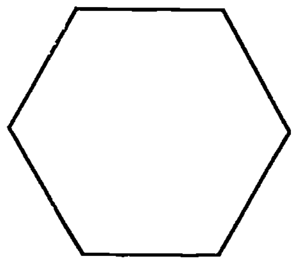

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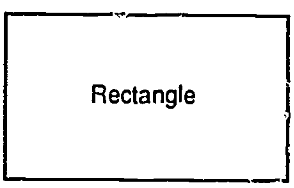

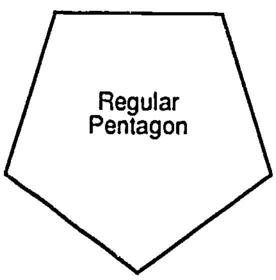

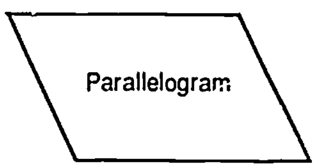

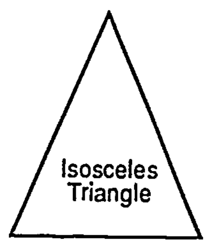

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Midpoint Madness Answers

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Thank you, attentive readers!

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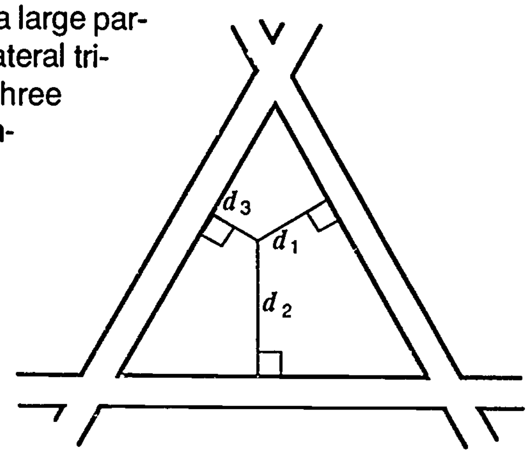
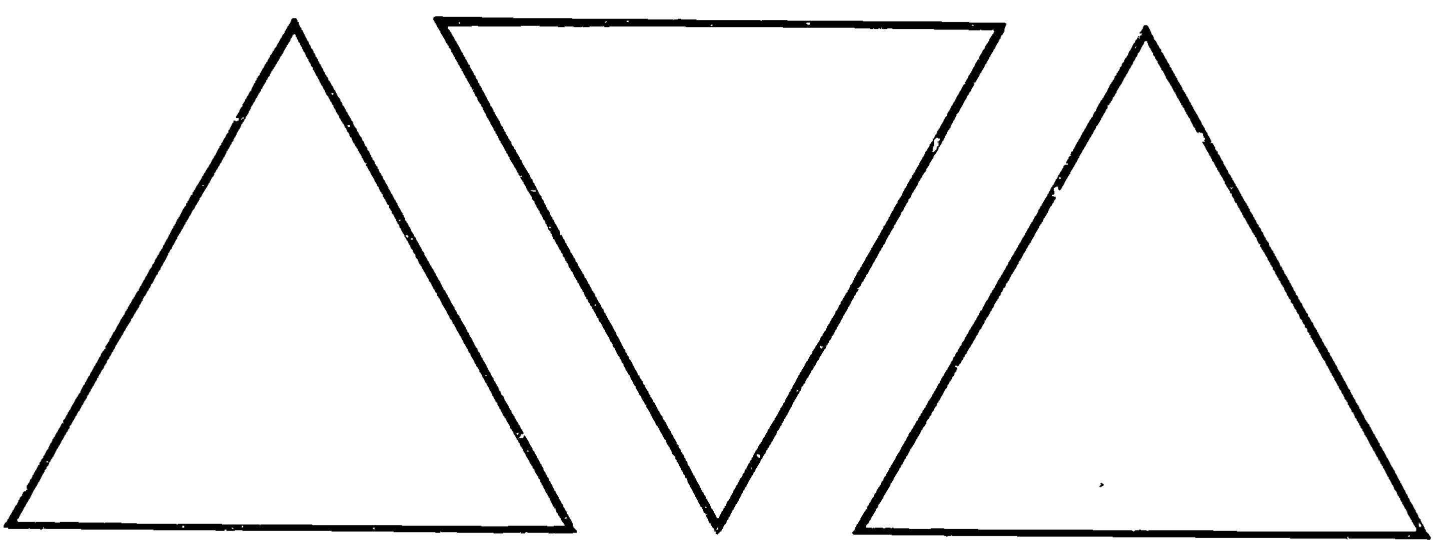
Minimum Distance

Stu Dent recently inherited a large parcel of land shaped like an equilateral triangle. The land's boundaries are three highways, intersecting to form the triangle's three vertices. However, for Stu to receive the land, the will required that he build a house on the triangular parcel of land and also that he build three roads from the house, each perpendicular to one of the three highways. Furthermore, the will required that the sum of the distances, $d_1 + d_2 + d_3$, from the house to the highways *must* be the **smallest** possible. Where should Stu put his house?



Let's Explore Some Possibilities for Stu

Using a ruler and protractor, locate possible points—housing sites—on or within the triangles below and construct the three roads from the house perpendicular to the highways. Measure and record the **sum** of the three distances under each triangle.

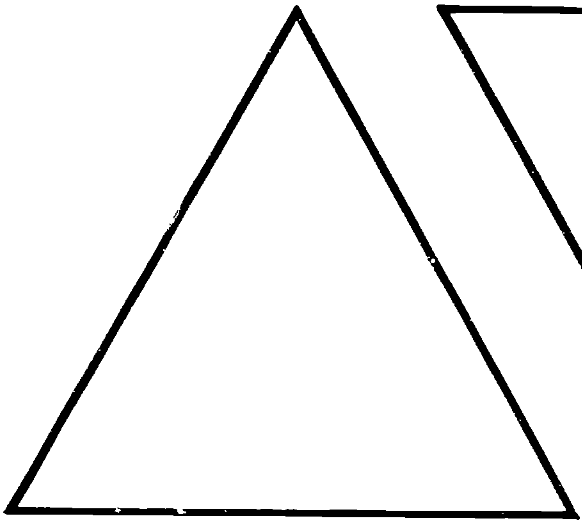


Sum = ___ Sum = ___ Sum = ___

The editors wish to thank Jerry L. Johnson, Western Washington University, Bellingham, WA 98225, for writing this issue of the *NCTM Student Math Notes.*

Continuing Your Explorations

Now try some additional points. Be sure to try special points, such as the intersection of the medians, a vertex, or a point on the edge of the parcel of land.



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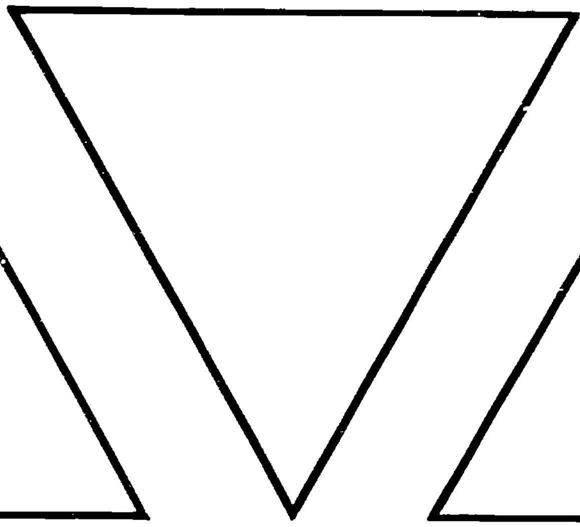

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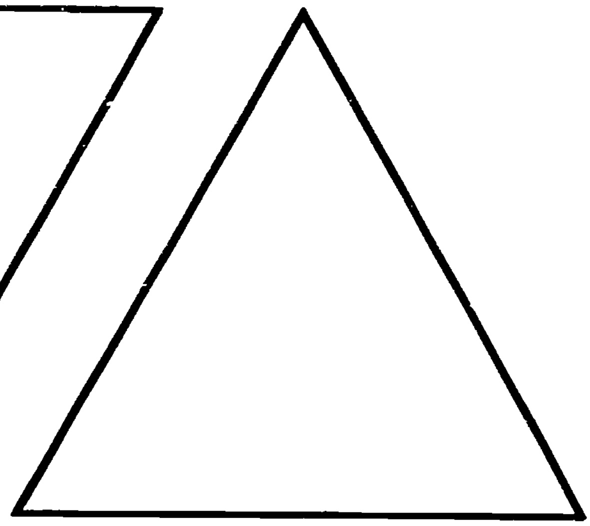


Sum = ___

- What is the sum of the distances whenever the point chosen is—

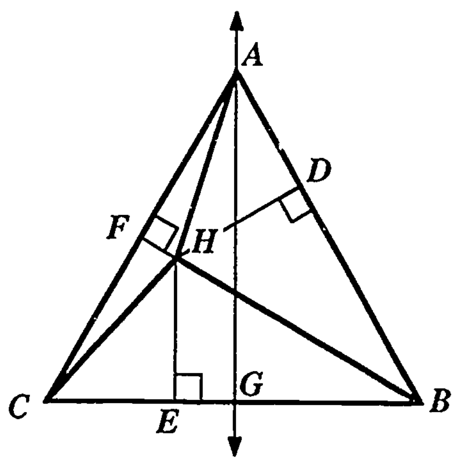
  within the triangle? ____________ on the edge of the triangle? ____________

  at a vertex of the triangle? ____________

- What appears to be true about all these sums? ____________

Verifying Your Discovery

The diagram at the right represents the conditions of the will. Point H was selected at random to represent the location of the house with $\overline{HD}$, $\overline{HE}$, and $\overline{HF}$ representing the three roads. Notice that the area of $\triangle ABC$ has been divided into three smaller triangles by $\overline{HC}$, $\overline{HB}$, and $\overline{HA}$, and that



area of $\triangle ABC$ = area of $\triangle BCH$ + area of $\triangle ACH$ + area of $\triangle ABH$.

Using the area formula, $A = (1/2)bh$, we have

$$\frac{1}{2}(BC)(AG) = \frac{1}{2}(BC)(HE) + \frac{1}{2}(AC)(HF) + \frac{1}{2}(AB)(HD).$$

Since $\triangle ABC$ is equilateral, the sides have equal measure. Therefore, by substituting BC for AC and AB, we have

$$\frac{1}{2}(BC)(AG) = \frac{1}{2}(BC)(HE) + \frac{1}{2}(BC)(HF) + \frac{1}{2}(BC)(HD)$$

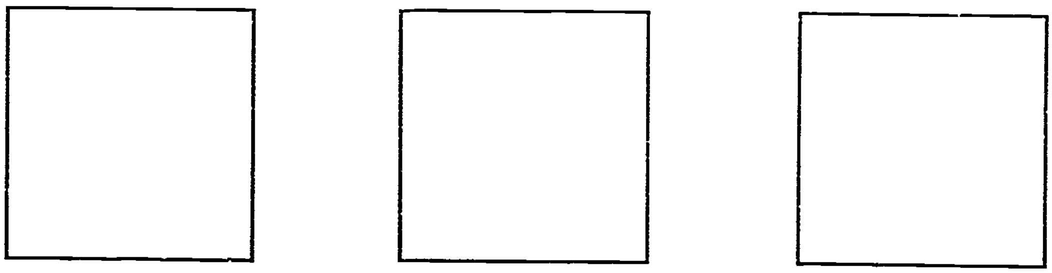
$$= \frac{1}{2}(BC)(HE + HF + HD).$$

By comparing both sides of this equation, we can see that $AG = HE + HF + HD$.

Therefore the sum of the three perpendicular segments, or roads, is a constant value. Regardless of where the house is located, the sum of the segments is equal to the length of the altitude of the original equilateral triangle, or land parcel. Now, construct an altitude in one of the triangles at the top of the page and measure its length to verify your discovery. Is its length equal to your sums? ____________

Further Explorations

If the parcel of land in the will had been in the shape of a square, where should Stu locate the house for the sum of the distances to the four sides to be a minimum?



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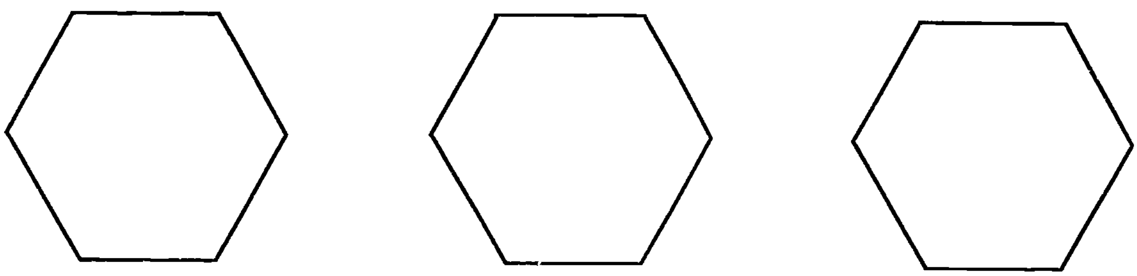

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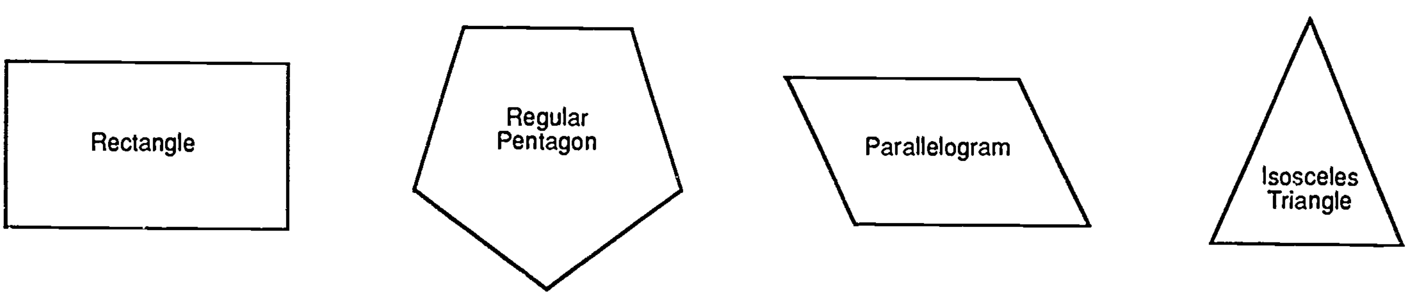

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On the Road with NCTM: Future Meeting Sites Are Announced

Don't pack your suitcase yet, but consider the possibilities: NCTM has announced its schedule of annual meetings and regional conferences for the next few years.

Aside from Chicago, the location of next month's 66th Annual Meeting, other locations for annual meetings in upcoming years are as follows:

- 67th: **Orlando**, 12–14 April 1989
- 68th: **Salt Lake City**, 18–21 April 1990
- 69th: **New Orleans**, 17–20 April 1991
- 70th: **Nashville**, 1–4 April 1992
- 71st: **Seattle**, 31 March–3 April 1993
- 72d: **Indianapolis**, 13–16 April 1994

Nashville, the home of country music, will be the site of the 70th Annual Meeting in 1992.

Here's the schedule of regional conferences for 1988–89:

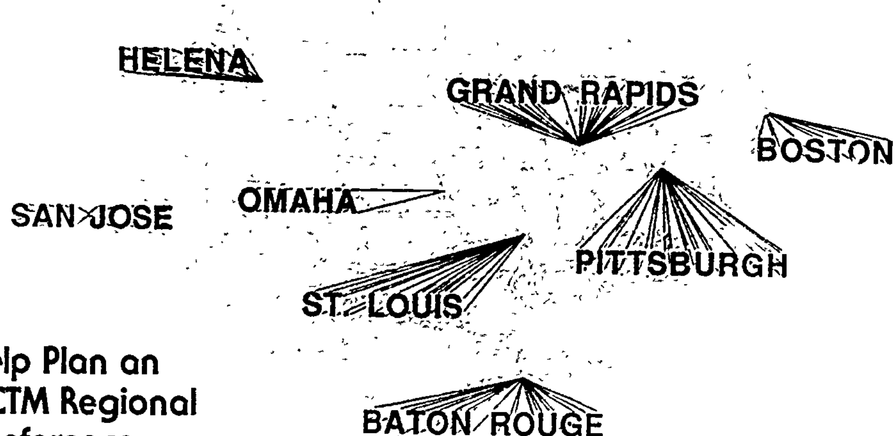
- **Pittsburgh**, 12–14 October
- **St. Louis**, 27–29 October
- **Baton Rouge**, 3–5 November
- **Boston**, 1–3 December
- **San Jose**, 23–25 February (1989)
- **Helena**, 2–4 March (1989)
- **Grand Rapids**, 9–11 March (1989)
- **Omaha**, 16–18 March (1989)

For those who keep multi-year appointment books, here are the locations of regional conferences in 1989–90:

- **Rapid City, South Dakota**, 5–7 October 1989
- **San Juan**, 12–14 October 1989
- **Denver**, 18–22 October 1989
- **Saskatoon, Saskatchewan**, 26–28 October 1989
- **Philadelphia**, 30 November–2 December 1989
- **Chattanooga**, 14–17 March 1990
- **Hamilton, Ontario**, 9–12 May 1990

Finally, for those who really plan ahead, the following is a list of meetings to be held in 1990–91:

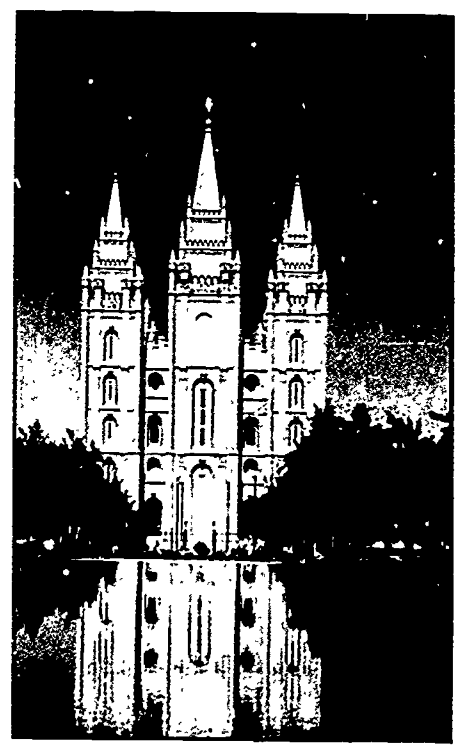
- **Honolulu**, Summer 1990
- **East Rutherford/Secaucus**, New Jersey, Fall 1990
- **Memphis**, Fall 1990
- **South Bend, Indiana**, Fall 1990
- **Wichita, Kansas**, Fall 1990
- **Madison, Wisconsin**, Fall/Winter 1990–91
- **Sacramento, California**, Spring 1991



Help Plan an NCTM Regional Conference

NCTM's Conventions and Conferences Committee invites you to participate in the plans being made for upcoming regional conferences. If you know a good speaker who can contribute to one of the meetings, or if you have any suggestions about types of sessions, scheduling, or other aspects of the meetings, please send in your ideas. Just ask for a recommendation form from the Headquarters Office—the form contains the names and addresses of program chairmen so you can easily send your ideas to the right person. (You should, however, send in your suggestions no later than 13 months before the meeting.)

For the locations of upcoming regional conferences, see the related article at the top of this page. Plan now to get involved in one or more of these important events.



This six-spired temple dominates Temple Square in Salt Lake City. (Courtesy of Salt Lake Convention and Visitors Bureau)

NCTM Arranges Calculator Sessions at NSTA and NAESP Conventions

NCTM members are scheduled to make presentations on the use of calculators in the mathematics classroom at the 1988 annual meetings of the National Science Teachers Association (NSTA) and the National Association of Elementary School Principals (NAESP).

On 7–10 April in St. Louis, Richard Lodholz of the Parkway School District in Creve Coeur, Missouri, will discuss "Improving Science Education in the Mathematics Class." He will focus on developmental lessons requiring an investigative approach that emphasizes process development and that is made possible by the technology of the hand-held calculator. The presentation reinforces NCTM's position statement urging the use of calculators in the classroom.

Also in line with NCTM's advocacy of calculators for all students will be the "brainstorming session" led by Kay Gilliland at the 1988 convention of the NAESP, to be held in San Francisco, California, 16–20 April. Gilliland, who is director of EQUALS in Computer Technology at the University of California at Berkeley, will explain NCTM's endorsement of calculators and outline the progress that various states are making in incorporating their use into the curriculum.

Go Around the World in 80 Ways with NCTM Travel Programs

It's time for your pop quiz in geography. Ready? Which of the following continents do NCTM tours and charter flights *not* reach?

a) Asia *b*) Europe *c*) Africa
d) Antarctica *e*) Australia

If you answered "*d*," then you're correct. NCTM does not arrange travel to Antarctica—yet. But you can see Asia, Africa, Europe, or Australia on one of the tours or charter flights arranged by the NCTM travel office.

What's your fancy? Rome? Mysterious Egypt? Do you like to follow your own whims when sightseeing? Or would you rather enjoy the company of a group in a guided tour? Either way, NCTM has something for you.

Charter Flights

If you prefer to set your own itinerary, then take advantage of the charter and discount flights that NCTM arranges through the National Center for Education Travel. You can fly from 62 cities in the U.S. to a host of cities around the globe. For information about round-trip airfares from points in the U.S. to such cities as Paris, Cairo, Tokyo, and Rio de Janeiro—and many others—fill out and mail the coupon below to George Marucci at the NCET.

Complete Tours

Would you rather leave the driving to us? Then get on board: Moscow! Kenya! Beijing! *Beijing?* Yes, you can even visit Beijing, the capital of mainland China that only recently opened its gates to Western travelers—and we'll add Hong Kong for some spice! Of course, you can also tour Rome—or Australia, New Zealand, eastern Europe, or Russia, to name just a few. Or if you want to *really* go for it, try the 26-day "Around the World" tour. It's guaranteed to expand your horizons. See the coupon below for a small sampling of the other tours that NCTM has put together for you this summer. If you haven't already received the colorful brochure with complete details, use the coupon labeled "Tours" to request it from Ed Nevins. Note that some tours are guided and others are not; but all tour packages include airfare, transfers to and from the airport, and hotel.

1988 Tours

Sample prices on tours:

- Athens (1 week), $799
- England, Ireland, Scotland (2 weeks), $1199
- Paris and French Riviera (2 weeks), $1199
- India (2 weeks), $1599
- Hong Kong and Beijing (2 weeks), $2099
- Around the World (26 days), $2999

Please send me your brochure with complete details.

Name ______________________

Address ______________________

____

Mail to Ed Nevins, NCTM Travel Coordinator, P.O. Box 366, Strasburg, VA 22657 (703/465-4211). Prices subject to change, but will be confirmed once bookings are made.
NCTM, NB 3/88

1988 Charter and Discount Flights

- From 62 cities in the U.S. to London, Glasgow, Shannon, Paris, Nice, Brussels, Amsterdam, Frankfurt, Munich, Hamburg, Düsseldorf, Vienna, Copenhagen, Geneva, Zurich, Rome, Milan, Athens, Tel Aviv, Cairo, Madrid, Malaga, Lisbon, Rio de Janeiro, Tokyo, Hong Kong, Taipei, Bangkok, Singapore, Beijing, Shanghai, Auckland, Sydney, Karachi, New Delhi, Bombay, eastern Europe.
- For NCTM members, their families, and friends

Name ______________________

Address ______________________

____

- For free brochure, send this coupon to NCET, 15109 Redgate Dr., Silver Spring, MD 20904 (301/384-5000).

NCTM, NB 3/88

student math notes

Statistical Decision Making

A panel of five experts rated the centers in the National Basketball Association (NBA) on five characteristics: aggressiveness, shooting range, teamwork, offense, and defense. The players were rated on each characteristic from 10 for excellent to 1 for poor. Table 1 gives the totals in each category for ten of the players.

Table 1

Player	Aggressiveness	Ranking	Shooting Range	Ranking	Teamwork	Ranking	Offense	Ranking	Defense	Ranking	Total of Rankings	Order
Brad Daugherty (Cleveland Cavaliers)	41		37		34	5	38		41			
Patrick Ewing (New York Knicks)	47		34		36	2	45		39			
Artis Gilmore (Chicago Bulls)	41		34		32	8	32		33			
Kareem Abdul Jabbar (Los Angeles Lakers)	33		38		48	1	48		35			
Bill Laimbeer (Detroit Pistons)	45		42		35	3	36		31			
Moses Malone (Washington Bullets)	49		37		27	10	46		36			
Akeem Olajuwon (Houston Rockets)	48		36		33	7	48		50			
Robert Parrish (Boston Celtics)	36		28		35	3	39		40			
Tree Rollins (Atlanta Hawks)	46		26		31	9	33		38			
Jack Sikma (Milwaukee Bucks)	33		43		34	5	36		28			

Using these results, who would you choose as the number-one center in the NBA? ____________

- A way to find the number-one center is to rank the players in each category from 1 (first) to 10 (last). The rank for teamwork would be 1—Jabbar (48); 2—Ewing (36); 3—Laimbeer, Parrish (35); 5—Daugherty, Sikma (34); 7—Olajuwon (33); 8—Gilmore (32); 9—Rollins (31); 10—Malone (27). Note that no 4 is listed because two players tie for number 3. Rank the other categories, total the ranks for each player, and order the players.
- Another way to select the best center is to determine who has the highest point total from the five characteristics. Find this total for each player and enter the results under "Total Points." Use those results to rank the players in the next column.
- Some people prefer to *weight* or count some of the categories more than others because they think that teamwork and offense, for example, are more important than the others. To find a weighted score, you might double the numbers for teamwork and offense and add them to the values in the other categories. For example, Daugherty's weighted total would be 41 + 37 + 2(34) + 2(38) + 41 = 263 (see table 2). Complete the column "Weighted Points" using this procedure. Now select two or more categories that you feel are more important, weight them, and complete the column "Your Weighted Points"; then rank the players.

Table 2

Player	Total Points	Ranking	Weighted Points	Ranking	Your Weighted Points	Ranking
Daugherty			263			
Ewing						
Gilmore						
Jabbar						
Laimbeer						
Malone						
Olajuwon						
Parrish						
Rollins						
Sikma						

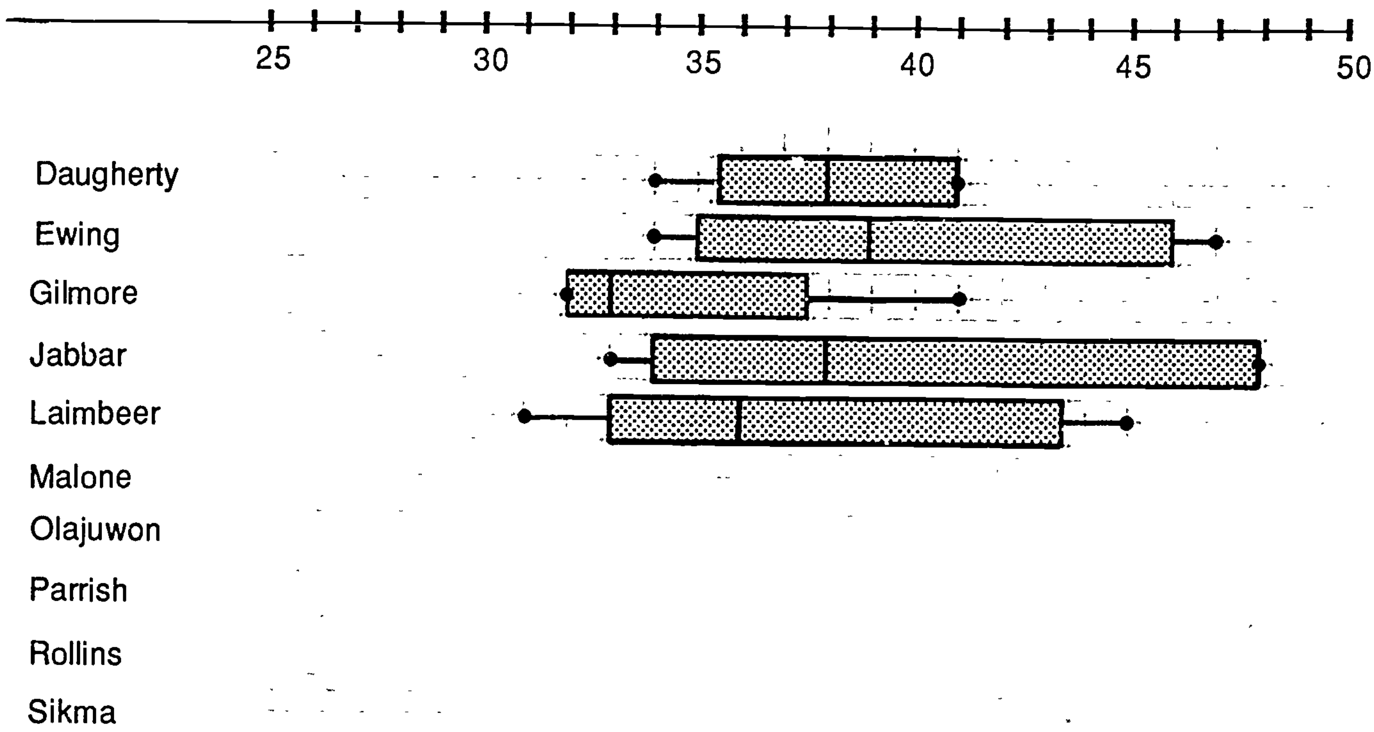
The editors wish to thank Gail Burrill, Whitnall High School, 5000 South 116 Street, Greenfield, WI 53228, for writing this issue of *NCTM Student Math Notes*.

Comparing These Methods

- How do the three methods compare? ______
- Which method did you like the best and why did you like it? ______
- Can you think of another way to rank the players? ______
- Who is your choice for the number-one center in the NBA? ______ Why? ______
- Who is the second best? ______ Why? ______
- If you were Malone's agent, which statistics would you use to stress that he was more valuable than Ewing? ______ Why? ______

Using Box-and-Whisker Plots

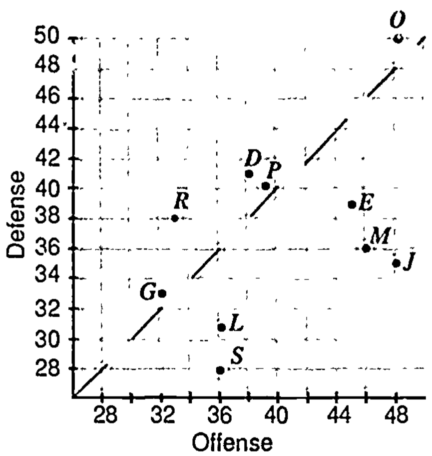
General managers might prefer to pay higher salaries to players who consistently have high ratings. A box-and-whisker plot of the ratings can help show consistency. To make a box plot, arrange the values for an individual player in order of size. Find the median, or middle, number and plot it on a number line. For example, Patrick Ewing's ratings are 47, 45, 39, 36, and 34, with a median of 39. Plot the lower quartile for the numbers, which is halfway between 36 and 34, or 35. Plot the upper quartile, which is 46, the average of 47 and 45. Draw a box from the lower-quartile value to the upper-quartile value (from 35 to 46). Draw a line, or whisker, from the box to the smallest rating, 34, and from the box to the largest value, 47. The line through the middle of the boxes is the middle, or median, rating. Complete the box plots below showing the range and the distribution of the five ratings given in table 1 for each player.



1. What is the median rating for Jack Sikma? ______
2. Which player has the lowest rating? ______
3. What players have more than half of their ratings above 37? ______
4. Which player has the highest median? ______
5. Which player is the most consistent in all his ratings? ______
6. Which player has the greatest variation in the ratings? ______
7. Which players have one rating that is much higher than their other ratings? ______
8. Sometimes the lowest score is discarded. Which player has four scores higher than any other player's four highest scores? ______

Using Scatterplots

Another way to look at the ratings is to look at combinations of the categories. Suppose you want to see who rates the highest in offense and defense only. A *scatterplot* of the offensive and defensive ratings for each player can help. For example, the point P corresponds to the ordered pair (39,40), the offensive and defensive ratings, respectively, for Robert Parrish.



Notice that the line $y = x$ has been drawn in on the scatterplot:

9. Where is the point for a player who has an equal offensive and defensive rating? ______
10. The highest possible rating for both offense and defense is represented by the point (50,50), since 50 is the maximum possible rating in any category. The point for the player who is best in both categories should be closest to the line $y = x$ in the upper-right corner. Which center has the highest rating for both offense and defense? ______
11. Which players rate higher in offense than defense? ______

    ______

    Where are the points representing those players? ______
12. What do the players above the line have in common? ______

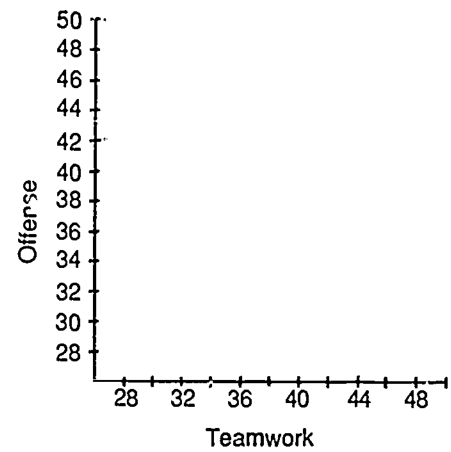
    ______

Complete the scatterplot for teamwork and offense.

13. Are the centers stronger in offense or in teamwork?

    ______ Why? ______

    ______
14. Which player is rated the highest in both offense and teamwork? ______
15. Sometimes statisticians make scatterplots of all possible pairs of categories and call this a matrix of the scatterplots. They look for any patterns they can find and to see which players rate the highest most often. How many different scatterplots would you have to make for a complete matrix of the NBA centers in which all pairs of categories are compared? ______



Using the Distance Formula

The distance formula can be used to find an algebraic solution for the player who has the highest rating in both offense and defense. The distance between two points (x_1,y_1) and (x_2,y_2) =

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

To find the distance from $P(39,40)$ for Robert Parrish to the point (50,50), the highest possible rating, we have

$$\begin{aligned}\sqrt{(50-39)^2 + (50-40)^2} &= \sqrt{11^2 + 10^2}\\ &= \sqrt{121 + 100}\\ &= \sqrt{221}.\end{aligned}$$

Robert Parrish is $\sqrt{221}$ units from the highest possible rating in both categories. You could simplify or use a calculator to approximate $\sqrt{221}$, but the numbers are easily compared in this form.

Find similar distances for each of the other players. Complete the chart for the comparison of both (offense, defense) and (teamwork, offense). Do the algebraic results using distance agree with your answers using the scatterplots?

______

Which method do you prefer? ______

Player	(Offense, Defense) Distance	(Teamwork, Offense) Distance
Daugherty		
Ewing		
Gilmore		
Jabbar		
Laimbeer		
Malone		
Olajuwon		
Parrish	$\sqrt{221}$	
Rollins		
Sikma		

Did You Know That . . .

- the formula for the number of possible combinations of n categories taken r at a time is $n!/(r!(n-r)!)$?
- Pat Ewing is currently the highest paid active NBA player, with a salary of $2,750,000 a year?
- the *Sporting News* 1987 Basketball Special rated Olajuwon as the number-one center, followed by Malone, Laimbeer, Ewing, and Sikma, in that order? The results were calculated by using the TENDEX formula: points plus rebounds plus assists plus blocked shots plus steals minus turnovers minus missed field-goal and free-throw attempts, with the total divided by minutes played and that quotient divided by the game pace.
- Statistics or data analysis is required for a college degree in many fields, including sociology, psychology, history, occupational therapy, accounting, economics, industrial engineering, and geography?
- the NBA center with the best 1986 free-throw shooting record is Jack Sikma of the Milwaukee Bucks, who made 87 percent of his free throws?
- the tallest NBA player is Manute Bol, who is 7 feet, 6 inches tall, and the shortest is Tyrone Bogues at 5 feet, 3 inches, both on the Washington Bullets?

Can You . . .

- find the number of different combinations if you compare three, four, or five of the categories from table 1 at a time?
- think of any other situations that could be analyzed in the same way as the NBA centers?
- rate the centers by finding the median for each category, assigning a plus 1 for a score above the median and a minus 1 for a score below the median, and then finding the total for each player?
- set up a similar rating scale for movies?

Answers

Player	Total of Rankings	Order	Total Points	Ranking	Weighted Points	Ranking	(Offense, Defense) Distance	(Teamwork, Offense) Distance
Daugherty	23	4	191	5	263	5	$\sqrt{225}$	$\sqrt{400}$
Ewing	20	2	201	3	282	3	$\sqrt{146}$	$\sqrt{221}$
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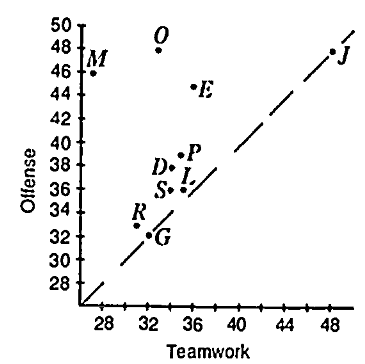
Jabbar	21	3	202	2	298	1	$\sqrt{229}$	$\sqrt{8}$
Laimbeer	26	6	189	6	260	6	$\sqrt{557}$	$\sqrt{421}$
Malone	24	5	195	4	268	4	$\sqrt{212}$	$\sqrt{545}$
Olajuwon	17	1	215	1	296	2	$\sqrt{4}$	$\sqrt{293}$
Parrish	28	7	178	7	252	7	$\sqrt{221}$	$\sqrt{346}$
Rollins	37	9	174	8	238	9	$\sqrt{433}$	$\sqrt{650}$
Sikma	32	8	174	8	244	8	$\sqrt{680}$	$\sqrt{452}$

Olajuwon is the number-one center according to two methods of ranking, and Jabbar is highest in the weighted ranking. Another answer could be acceptable also if the r for ranking can be explained.

1. 34
2. Tree Rollins
3. Daugherty, Ewing, Olajuwon, Jabbar
4. Olajuwon
5. Daugherty
6. Malone
7. Gilmore and Rollins
8. Daugherty
9. On the line $y = x$
10. Olajuwon
11. Laimbeer, Sikma, Ewing, Malone, Jabbar. Below the line $y = x$
12. They rate higher in defense than offense.
13. Offense, all points below the line
14. Jabbar
15. 10



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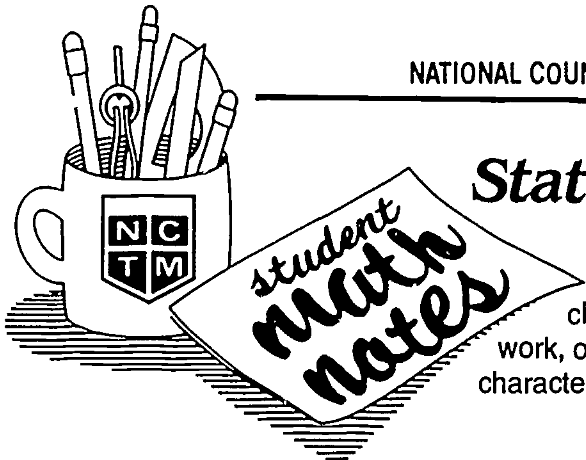


Statistical Decision Making

A panel of five experts rated the centers in the National Basketball Association (NBA) on five characteristics: aggressiveness, shooting range, teamwork, offense, and defense. The players were rated on each characteristic from 10 for excellent to 1 for poor. Table 1 gives the totals in each category for ten of the players.

Table 1

Player	Aggressiveness	Ranking	Shooting Range	Ranking	Teamwork	Ranking	Offense	Ranking	Defense	Ranking	Total of Rankings	Order
Brad Daugherty (Cleveland Cavaliers)	41		37		34	5	38		41			
Patrick Ewing (New York Knicks)	47		34		36	2	45		39			
Artis Gilmore (Chicago Bulls)	41		34		32	8	32		33			
Kareem Abdul Jabbar (Los Angeles Lakers)	33		38		48	1	48		35			
Bill Laimbeer (Detroit Pistons)	45		42		35	3	36		31			
Moses Malone (Washington Bullets)	49		37		27	10	46		36			
Akeem Olajuwon (Houston Rockets)	48		36		33	7	48		50			
Robert Parrish (Boston Celtics)	36		28		35	3	39		40			
Tree Rollins (Atlanta Hawks)	46		26		31	9	33		38			
Jack Sikma (Milwaukee Bucks)	33		43		34	5	36		28			

Using these results, who would you choose as the number-one center in the NBA? ____________

- A way to find the number-one center is to rank the players in each category from 1 (first) to 10 (last). The rank for teamwork would be 1—Jabbar (48); 2—Ewing (36); 3—Laimbeer, Parrish (35); 5—Daugherty, Sikma (34); 7—Olajuwon (33); 8—Gilmore (32); 9—Rollins (31); 10—Malone (27). Note that no 4 is listed because two players tie for number 3. Rank the other categories, total the ranks for each player, and order the players.
- Another way to select the best center is to determine who has the highest point total from the five characteristics. Find this total for each player and enter the results under "Total Points." Use those results to rank the players in the next column.
- Some people prefer to *weight* or count some of the categories more than others because they think that teamwork and offense, for example, are more important than the others. To find a weighted score, you might double the numbers for teamwork and offense and add them to the values in the other categories. For example, Daugherty's weighted total would be 41 + 37 + 2(34) + 2(38) + 41 = 263 (see table 2). Complete the column "Weighted Points" using this procedure. Now select two or more categories that you feel are more important, weight them, and complete the column "Your Weighted Points"; then rank the players.

Table 2

Player	Total Points	Ranking	Weighted Points	Ranking	Your Weighted Points	Ranking
Daugherty			263			
Ewing						
Gilmore						
Jabbar						
Laimbeer						
Malone						
Olajuwon						
Parrish						
Rollins						
Sikma						

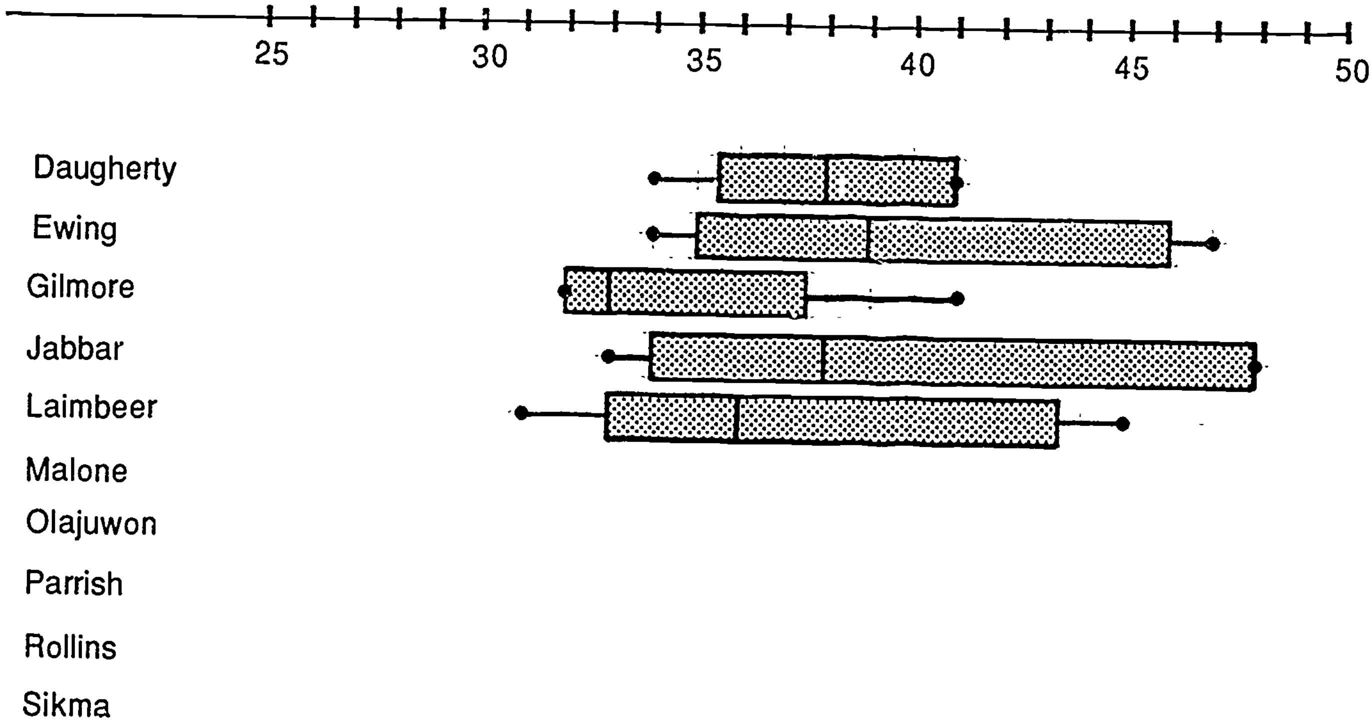
The editors wish to thank Gail Burrill, Whitnall High School, 5000 South 116 Street, Greenfield, WI 53228, for writing this issue of *NCTM Student Math Notes*.

Comparing These Methods

- How do the three methods compare? ______________________________
- Which method did you like the best and why did you like it? ______________________________
- Can you think of another way to rank the players? ______________________________
- Who is your choice for the number-one center in the NBA? ____________ Why? ______________
- Who is the second best? ____________ Why? ______________________________
- If you were Malone's agent, which statistics would you use to stress that he was more valuable than Ewing? ________
  Why? ______________________________

Using Box-and-Whisker Plots

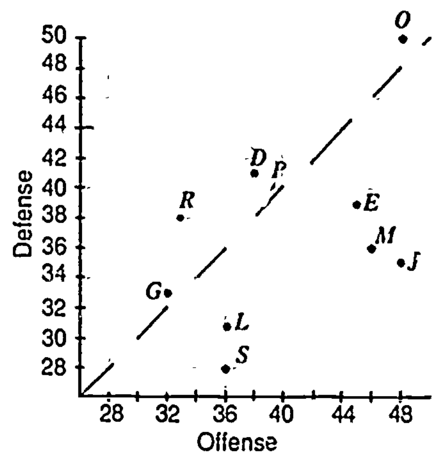
General managers might prefer to pay higher salaries to players who consistently have high ratings. A box-and-whisker plot of the ratings can help show consistency. To make a box plot, arrange the values for an individual player in order of size. Find the median, or middle, number and plot it on a number line. For example, Patrick Ewing's ratings are 47, 45, 39, 36, and 34, with a median of 39. Plot the lower quartile for the numbers, which is halfway between 36 and 34, or 35. Plot the upper quartile, which is 46, the average of 47 and 45. Draw a box from the lower-quartile value to the upper-quartile value (from 35 to 46). Draw a line, or whisker, from the box to the smallest rating, 34, and from the box to the largest value, 47. The line through the middle of the boxes is the middle, or median, rating. Complete the box plots below showing the range and the distribution of the five ratings given in table 1 for each player.



1. What is the median rating for Jack Sikma? ______________________________
2. Which player has the lowest rating? ______________________________
3. What players have more than half of their ratings above 37? ______________________________
4. Which player has the highest median? ______________________________
5. Which player is the most consistent in all his ratings? ______________________________
6. Which player has the greatest variation in the ratings? ______________________________
7. Which players have one rating that is much higher than their other ratings? ______________________________
8. Sometimes the lowest score is discarded. Which player has four scores higher than any other player's four highest scores? ______________________________

Using Scatterplots

Another way to look at the ratings is to look at combinations of the categories. Suppose you want to see who rates the highest in offense and defense only. A *scatterplot* of the offensive and defensive ratings for each player can help. For example, the point ***P*** corresponds to the ordered pair (39,40), the offensive and defensive ratings, respectively, for Robert Parrish.



Notice that the line $y = x$ has been drawn in on the scatterplot:

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Complete the scatterplot for teamwork and offense.

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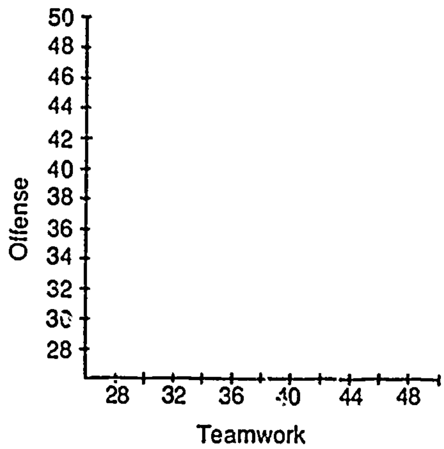

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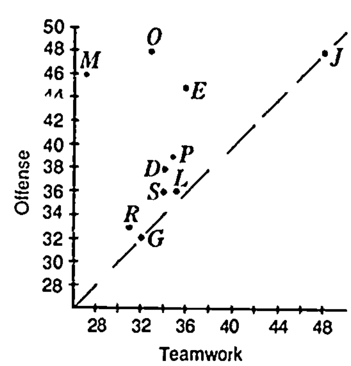

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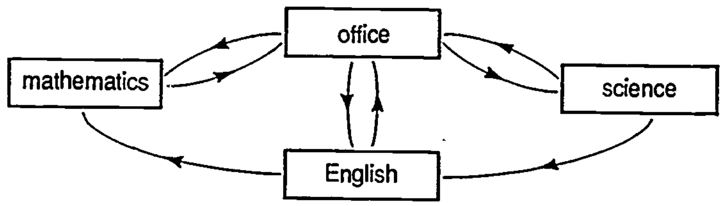
Directed Graphs

At U-Rah High School the intercom lines are connected according to the following graph:



This graph is called a directed graph or *digraph*, with the arrows indicating the direction of the flow. Another method to show the same information is to use ordered pairs, where m = mathematics, e = English, o = office, and s = science.

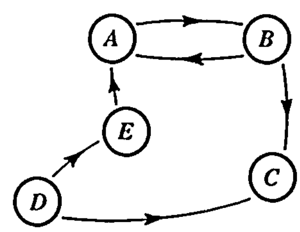
$$(m,o), (o,m), (o,e), (o,s), (s,o), (s,e), (e,o), (e,m)$$

A third method used to show this information or to record the ordered pairs is in an array called a matrix.

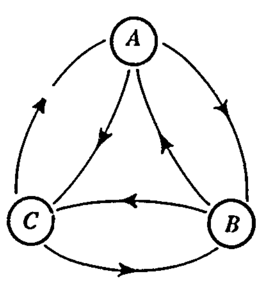
	m	s	o	e
m	0	0	1	0
s	0	0	1	1
o	1	1	0	1
e	1	0	1	0

Each entry in the array corresponds to the number of single arrows connecting consecutive vertices. A 1 appears in the (e,m) cell because an arrow connects English and mathematics.

1. What is the difference between (o,s) and (s,o)? ____________
2. Is it possible for the mathematics department to place a direct call to the English department? ____
3. How could the mathematics department call the English department? ____________
4. What does the 1 corresponding to the pair (e,o) in the matrix mean? ____________
5. In the diagram at the right, the symbol A $\rightarrow$ B means that team A defeats team B. List the ordered pairs that describe the digraph. ____________



6. Write the matrix for the following digraph.



The editors wish to thank Jack and Gail Burrill, Whitnall High School, 5000 South 116 Street, Greenfield, WI 53228, for writing this issue of *NCTM Student Math Notes*.

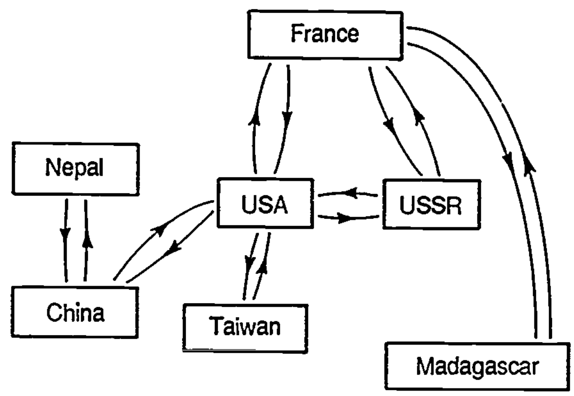
Directed Graphs—Continued

7. From information obtained in the digraph, use the ordered pairs in number 5 to create a matrix. For example, in number 5 we see the ordered pair (E,A). From the digraph we know that E defeated A. In the matrix in the row marked E and under column A, we assign a 1. If E had not defeated A, we would have assigned a 0. Complete the matrix at the right.

	A	B	C	D	E
A	0	1			
B			1		0
C					
D					
E	1				

8. Use the matrix in number 7 to answer the following questions:
   a) What is the win-loss record of team A? ______
   b) If A is to play D, who do you predict to win? ______
   c) What is the total number of wins in the conference? ______
   d) Which team has the best record? ______ ______
   e) Which team has the worst record? ______

At the right is a digraph concerning the diplomatic relations among certain governments.

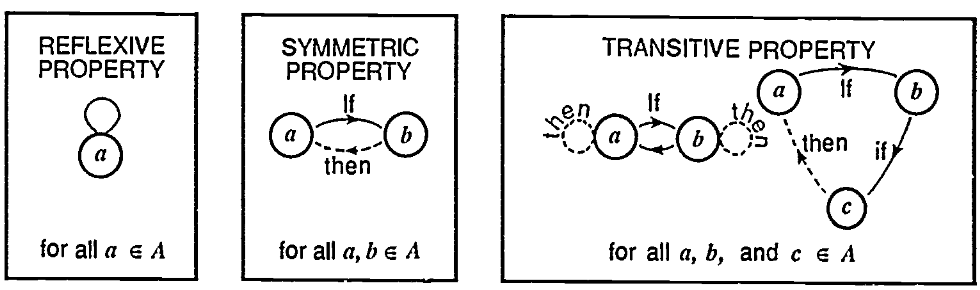


9. How would the USSR communicate with Madagascar? ______
10. How would Taiwan communicate with China? ______
11. Which countries are most isolated? ______

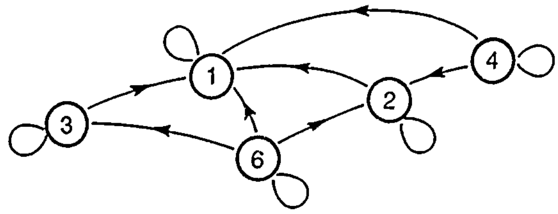
    Which country has the most diplomatic relations? ______
12. If every country communicated with every other country, how many ordered pairs would result? ______

Extending Your Discovery

Digraphs can be used to study sets of ordered pairs concerning specific information, such as telephone networks or tournament schedules. Some digraphs have special mathematical properties. In fact, these properties can be defined using digraphs. Let A be a set.



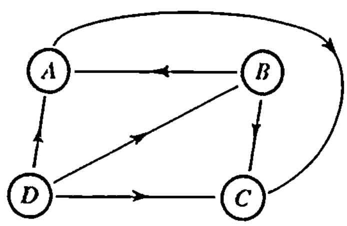
For example, the relation "is a multiple of" on the set {1, 2, 3, 4, 6} results in the digraph at the right. This relation has both the reflexive and transitive properties. It does not have the symmetric property because 6 is a multiple of 3 but 3 is not a multiple of 6.



$a \rightarrow b$ means a is a multiple of b

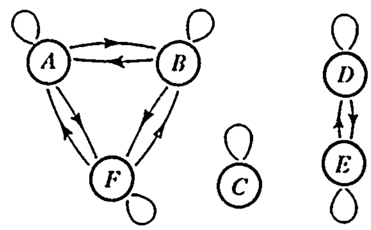
13. Which properties listed on page 2 do the following relations have on the given sets? (The corresponding digraphs are pictured.)

a) "Is taller than" on the set of children listed below:



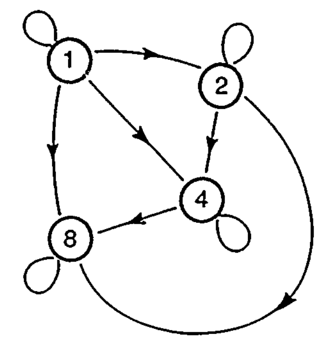
Name	Height
Anna	85 cm
Barta	90 cm
Cara	72 cm
Dona	110 cm

b) "Is in the same grade as" on the set of children listed below:



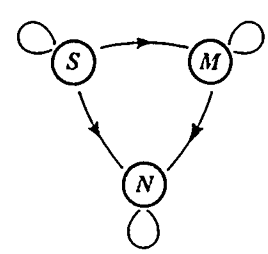
Name	Grade
Anna	1
Bai	1
Casey	3
David	2
Evan	2
Francis	1

c) "Is a factor of" on the set {1, 2, 4, and 8}

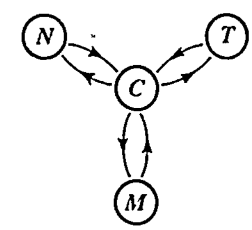


14. Decide whether each relation at the right has reflexive, symmetric, or transitive properties.

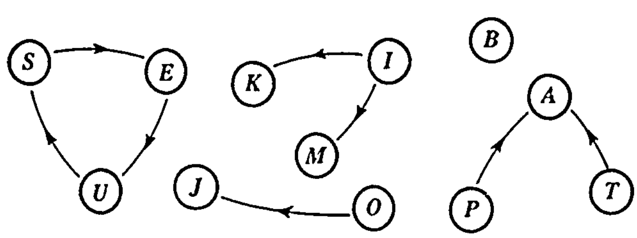
a)



b)



15. In the digraph below, involving twelve people, part of the arrows are drawn on the relation "is the sister of." Add any other arrows that can be deduced.



16. In the digraph pictured below, $a \twoheadrightarrow b$ means a is the mother of b.

a) How is a related to c? ____________________

b) How is b related to a? ____________________

c) How is c related to b? ____________________

d) How is c related to a? ____________________

17. Make a digraph for each of these sets of ordered pairs and decide whether each relation has reflexive, symmetric. or transitive properties.

a) {(1,1), (1,2), (1,3), (2,1), (2,3), (2,2), (3,3), (3,1), (3,2)}

b) {(1,2), (2,3), (1,3), (3,1), (2,2), (3,2)}

c) {(1,1), (1,3), (2,3), (3,2), (3,1), (2,4), (3,4), (4,4)}

18. Decide whether each relation has reflexive, symmetric, or transitive properties.

a)

	1	2	3
1	0	1	1
2	1	1	0
3	0	1	1

b)

	1	2	3
1	1	1	1
2	1	1	1
3	1	1	1

Can You . . .

- find the number of intercom lines you would need to connect each of five people to each other?
- find a relation that has the reflexive and symmetric properties but not the transitive properties?
- find a relation that has the symmetric property but not the transitive property?

Did You Know That . . .

- digraphs are derived from the work done by Leonhard Euler and the Königsberg bridge problem?
- digraphs can be used to find the critical path showing the length and sequence of the activities in a project?
- digraphs were used to help develop the System Flow Plan for the engine of the Polaris submarine?
- digraphs can be used to trace the passing of DDT through the ecological food chain?
- the square of a matrix for a digraph will give the number of two-edged paths that connect any two vertices; the cube will give the number of three-edged paths that connect any two vertices?

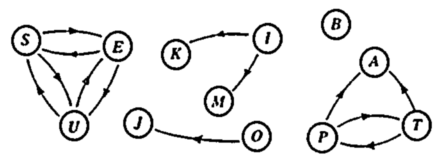
Answers

1. (o,s) indicates that the office can call the science department directly; (s,o) indicates that the science department can call the office directly.
2. No
3. One possibility is (m,o) and (o,e).
4. An arrow connects the English department to the office.
5. (A,B), (B,A), (B,C), (D,C), (D,E), (E,A)
6.

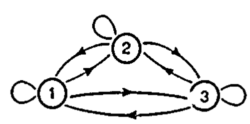
	A	B	C
A	0	1	1
B	1	0	1
C	1	1	0

7.

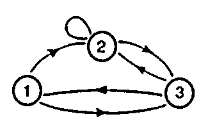
	A	B	C	D	E
A	0	1	0	0	0
B	1	0	1	0	0
C	0	0	0	0	0
D	0	0	1	0	1
E	1	0	0	0	0

8. (*a*) Won 1, lost 2; (*b*) Because A has won one game while D has won two, you might expect D to win. Also D defeated E and E defeated A, so that you might expect D to win; (*c*) 6; (*d*) B and D have each won two games. (e) C has won no games.
9. One way is through France to Madagascar
10. One way is through USA to China
11. Nepal, Madagascar; USA
12. 42
13. a) The transitive property
    b) The reflexive, symmetric, and transitive properties
    c) The reflexive and transitive properties
14. a) Reflexive and transitive
    b) Symmetric
15. 

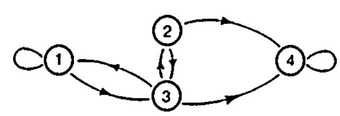
16. *a*) *a* is the grandmother of *c*.
    b) *b* is the daughter of *a*.
    c) *c* is the child of *b*.
    d) *c* is the grandchild of *a*.
17. *a*) The relation is reflexive, symmetric, and transitive.



b) None



c) None



18. (*a*) None; (*b*) all

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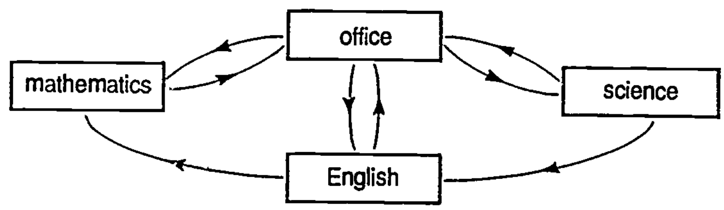
Directed Graphs

At U-Rah High School the intercom lines are connected according to the following graph:



This graph is called a directed graph or *digraph,* with the arrows indicating the direction of the flow. Another method to show the same information is to use ordered pairs, where m = mathematics, e = English, o = office, and s = science.

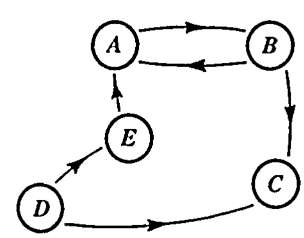
$$(m,o), (o,m), (o,e), (o,s), (s,o), (s,e), (e,o), (e,m)$$

A third method used to show this information or to record the ordered pairs is in an array called a matrix.

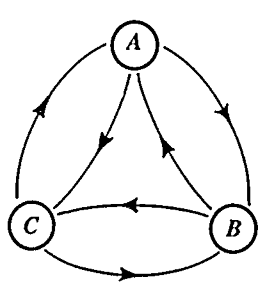
	m	s	o	e
m	0	0	1	0
s	0	0	1	1
o	1	1	0	1
e	1	0	1	0

Each entry in the array corresponds to the number of single arrows connecting consecutive vertices. A 1 appears in the (e,m) cell because an arrow connects English and mathematics.

1. What is the difference between (o,s) and (s,o)? ______
2. Is it possibie for the mathematics department to place a direct call to the English department? ______
3. How could the mathematics department call the English department? ______
4. What does the 1 corresponding to the pair (e,o) in the matrix mean? ______
5. In the diagram at the right, the symbol A $\rightarrow$ B means that team A defeats team B. List the ordered pairs that describe the digraph. ______



6. Write the matrix for the following digraph.



The editors wish to thank Jack and Gail Burrill, Whitnall High School, 5000 South 116 Street, Greenfield, WI 53228, for writing this issue of *NCTM Student Math Notes.*

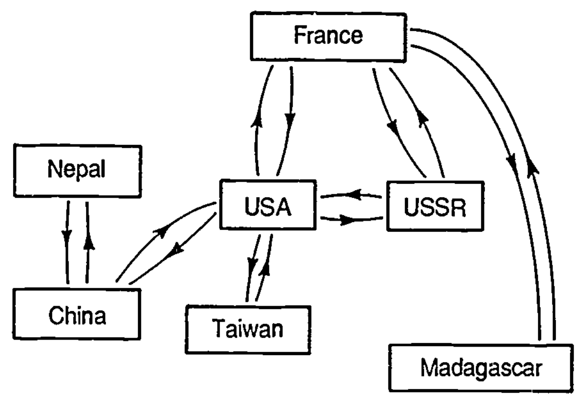
Directed Graphs—Continued

7. From information obtained in the digraph, use the ordered pairs in number 5 to create a matrix. For example, in number 5 we see the ordered pair (E,A). From the digraph we know that E defeated A. In the matrix in the row marked E and under column A, we assign a 1. If E had not defeated A, we would have assigned a 0. Complete the matrix at the right.

	A	B	C	D	E
A	0	1			
B			1		0
C					
D					
E	1				

8. Use the matrix in number 7 to answer the following questions:
   a) What is the win-loss record of team A? ______
   b) If A is to play D, who do you predict to win? ______
   c) What is the total number of wins in the conference? ______
   d) Which team has the best record? ______
   e) Which team has the worst record? ______

At the right is a digraph concerning the diplomatic relations among certain governments.

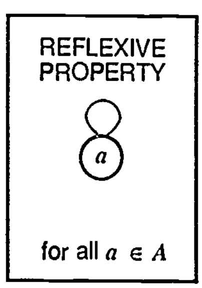


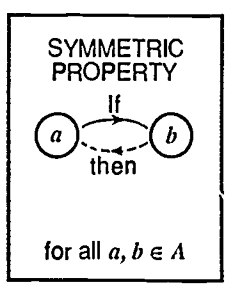
9. How would the USSR communicate with Madagascar? ______
10. How would Taiwan communicate with China? ______
11. Which countries are most isolated? ______

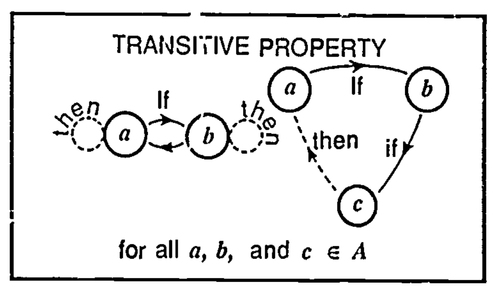
    Which country has the most diplomatic relations? ______
12. If every country communicated with every other country, how many ordered pairs would result? ______

Extending Your Discovery

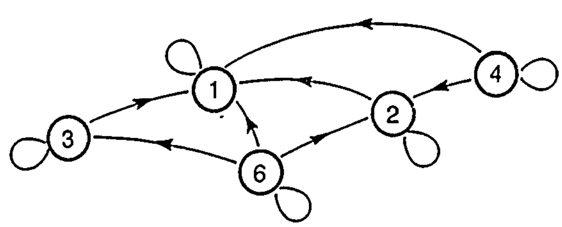
Digraphs can be used to study sets of ordered pairs concerning specific information, such as telephone networks or tournament schedules. Some digraphs have special mathematical properties. In fact, these properties can be defined using digraphs. Let A be a set.







For example, the relation "is a multiple of" on the set {1, 2, 3, 4, 6} results in the digraph at the right. This relation has both the reflexive and transitive properties. It does not have the symmetric property because 6 is a multiple of 3 but 3 is not a multiple of 6.

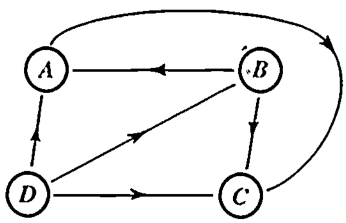


$a \rightarrow b$ means a is a multiple of b

Extending Your Discovery—Continued

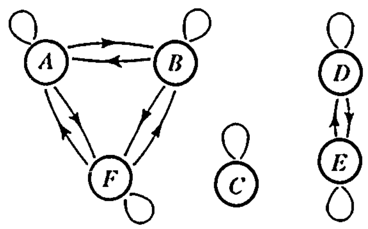
13. Which properties listed on page 2 do the following relations have on the given sets? (The corresponding digraphs are pictured.)

a) "Is taller than" on the set of children listed below:



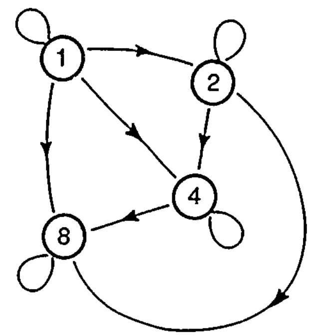
Name	Height
Anna	85 cm
Barta	90 cm
Cara	72 cm
Dona	110 cm

b) "Is in the same grade as" on the set of children listed below:



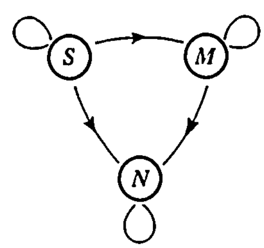
Name	Grade
Anna	1
Bai	1
Casey	3
David	2
Evan	2
Francis	1

c) "Is a factor of" on the set {1, 2, 4, and 8}

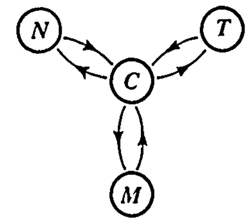


14. Decide whether each relation at the right has reflexive, symmetric, or transitive properties.

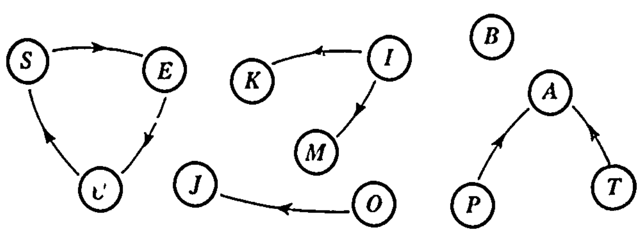
a)



b)



15. In the digraph below, involving twelve people, part of the arrows are drawn on the relation "is the sister of." Add any other arrows that can be deduced.



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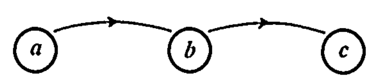

a) How is a related to c? ______

b) How is b related to a? ______

c) How is c related to b? ______

d) How is c related to a? ______

17. Make a digraph for each of these sets of ordered pairs and decide whether each relation has reflexive, symmetric, or transitive properties.

a) {(1,1), (1,2), (1,3), (2,1), (2,3), (2,2), (3,3), (3,1), (3,2)}

b) {(1,2), (2,3), (1,3), (3,1), (2,2), (3,2)}

c) {(1,1), (1,3), (2,3), (3,2), (3,1), (2,4), (3,4), (4,4)}

18. Decide whether each relation has reflexive, symmetric, or transitive properties.

a)

	1	2	3
1	0	1	1
2	1	1	0
3	0	1	1

b)

	1	2	3
1	1	1	1
2	1	1	1
3	1	1	1

Can You . . .

- find the number of intercom lines you would need to connect each of five people to each other?
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- find a relation that has the symmetric property but not the transitive property?

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Answers

1. (o,s) indicates that the office can call the science department directly; (s,o) indicates that the science department can call the office directly.
2. No
3. One possibility is (m,o) and (o,e).
4. An arrow connects the English department to the office.
5. (A,B), (B,A), (B,C), (D,C), (D,E), (E,A)
6.

	A	B	C
A	0	1	1
B	1	0	1
C	1	1	0

7.

	A	B	C	D	E
A	0	1	0	0	0
B	1	0	1	0	0
C	0	0	0	0	0
D	0	0	1	0	1
E	1	0	0	0	0

8. (*a*) Won 1, lost 2; (*b*) Because A has won one game while D has won two, you might expect D to win. Also D defeated E and E defeated A, so that you might expect D to win; (*c*) 6; (*d*) B and D have each won two games. (*e*) C has won no games.
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10. One way is through USA to China
11. Nepal, Madagascar; USA
12. 42
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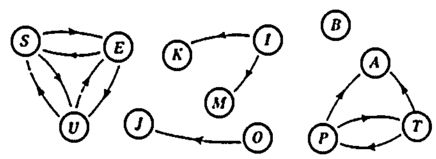
    b) The reflexive, symmetric, and transitive properties
    c) The reflexive and transitive properties
14. *a*) Reflexive and transitive
    b) Symmetric
15.



16. *a*) a is the grandmother of c.
    b) b is the daughter of a.
    c) c is the child of b.
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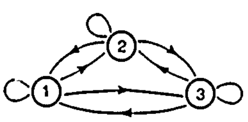
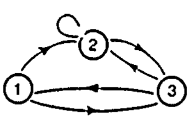
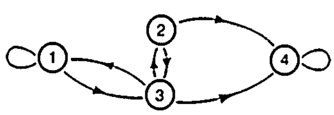
17. *a*) The relation is reflexive, symmetric, and transitive.



b) None



c) None



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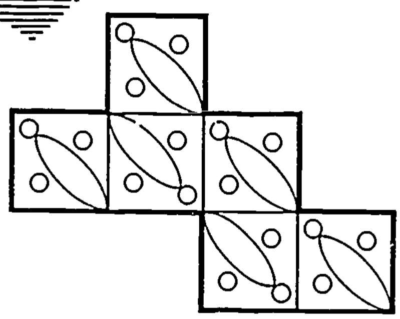
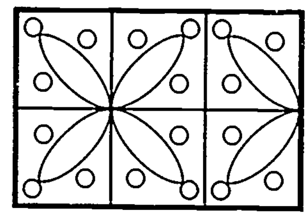
student math notes

Investigating Perimeter and Area

DeCora and DeCore, interior decorators, are constructing mosaic designs using colored ceramic square tiles. Each square is the same size, and the length of a side is one unit. With six squares, they created the following designs:



Design 1



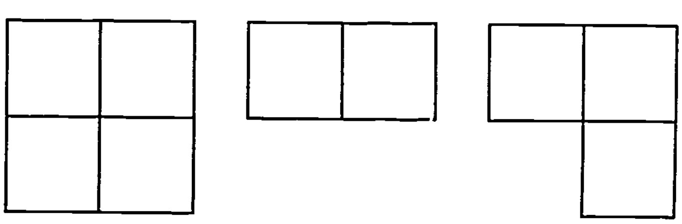
Design 2

It is interesting to note that even though the areas are the same, the perimeters are different. Find the perimeter of each figure to see how they differ. To determine the price of a mosaic, DeCora and DeCore charge $8 for each tile and $5 per unit of length for framing (1 unit of length = the length of one side of the tile).

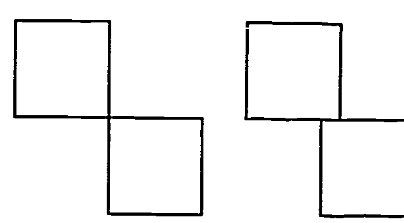
- Find the total cost of the tile and framing for each mosaic shown above.

Design 1 = ____________ Design 2 = ____________

DeCora and DeCore often have to explain to customers that designs with the same area may have different total costs, so they need a good understanding of the relationship between the perimeter and area of polygonal figures. The following activities offer an opportunity to explore the relationship between perimeter and area using arrangements of squares. In each arrangement, neighboring squares must share a common side.



(These arrangements can be used.)



(These cannot be used.)

Investigating Perimeter

You will need grid paper and fifty to one hundred square tiles or squares cut from construction paper. Let the length of the side of a square be one unit. Note that the area of each arrangement is equal to the number of squares. Work with a group of classmates to investigate the perimeter of certain arrangements found by putting squares together.

The editors wish to thank Judy Mumme, mathematics department of the University of California at Santa Barbara, for writing this issue of *NCTM Student Math Notes*.

Investigating Perimeter—Continued

- Verify the results in table 1 for one, two, and three squares.
- Explore different arrangements of squares to determine the perimeter for each.
- Explore the range of perimeters possible for each number of squares to find the arrangements that produce the maximum and minimum perimeters.
- Use grid paper to sketch the arrangements for the maximum and minimum perimeters.
- Record the maximum and minimum perimeters for the area arrangements in table 1.
- Look for patterns in the table and on the grid paper to see if you can predict the maximum and minimum perimeter of an arrangement. You should extend table 1.

Table 1

Area (No. of Sq.)	Maximum Perimeter	Minimum Perimeter
1	4	4
2	6	6
3	8	8
4	10	
5		10
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		

1. Can you find an arrangement for which the perimeter is an odd number? If so, draw the arrangement. If not, explain. ______

2. Predict the *maximum* perimeter of arrangements of the following numbers of squares:

   40 ______ 75 ______ 100 ______

3. Make a conjecture for finding the *maximum* perimeter of arrangements of any number of squares. ______

4. Now, try to predict the *minimum* perimeter of arrangements of the following numbers of squares:

   36 ______ 48 ______ 55 ______

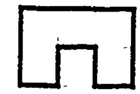
5. Make a conjecture for finding the *minimum* perimeter of arrangements of any number of squares. ______

Investigating Areas

Using the same rule for arranging squares, explore the minimum and maximum *area* of an arrangement of squares with a *given perimeter*.

- The arrangement shown in the grid has a perimeter of 10. Outline five other arrangements that have a perimeter of 10. Indicate the area of each arrangement on the grid.

- The arrangement shown on this grid has a perimeter of 12. Outline as many other arrangements as you can that have a perimeter of 12. Indicate the area of each.



- Continue to explore different arrangements of squares that produce a given perimeter and determine the area of each arrangement in order to find arrangements that produce the *minimum* and *maximum* areas. Record these values in table 2.

Table 2

Perimeter	Minimum Area	Maximum Area
4	1	1
6	2	2
8	3	4
10	4	
12		9
14		
16		
18		
20		
22		
24		
26		

6. Predict the *minimum* area for the following perimeters:

   30 ____________ 36 ____________ 44 ____________

7. Make a conjecture for finding the *minimum* area.

   ____

   ____

   ____

8. Predict the *maximum* area for the following perimeters:

   30 ____________ 36 ____________ 42 ____________

9. Make a conjecture for finding the *maximum* area.

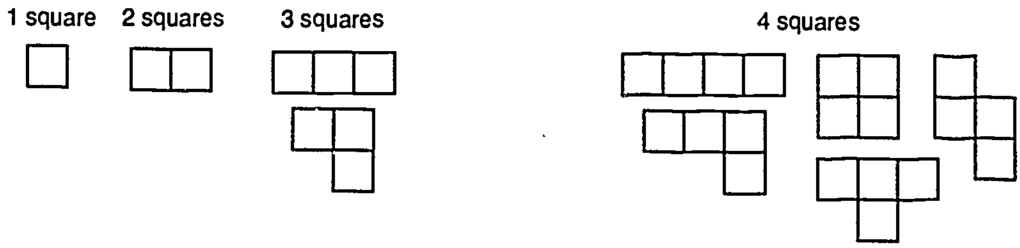
   ____

   ____

   ____

Did You Know That . . .

- using the same rule as before, only one way exists to arrange one square, one way to arrange two squares, two ways to arrange three squares, and five ways to arrange four squares?



- you can add squares to an arrangement to increase the area and decrease the perimeter?
- this investigation can be extended to three dimensions using cubes to explore the relationship between surface area and volume?
- design and packaging engineers also have to consider the relationship between perimeter and area in maximizing profit while minimizing cost. Similar concerns face civil engineers and housing developers when they lay out streets and lot designs.

Can You . . .

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Answers

Page 1

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Table 1

Max: 4, 6, 8, 10, 12, 14, 16, 18, . . .

Min: 4, 6, 8, 8, 10, 10, 12, 12, 12, 14, 14, 14, 16, 16, 16, 16, 18, 18, 18, 18, 20, 20, 20, 20, 20, . . .

1. No, the perimeter of a square is an even number; if a square is added to an arrangement, the perimeter stays the same or increases or decreases by a multiple of 2.
2. 82, 152, 202
3. Twice the number of squares plus 2, $P = 2n + 2$
4. 24, 28, 30
5. Answers will vary with sophistication: Let a be the number of squares; let g be the greatest integer in $\sqrt{a}$ (the whole-number square root). If $\sqrt{a} = g$, then $p = 4\sqrt{a}$. If $a \leq g^2 + g$, then $p = 4g + 2$. If $a > g^2 + g$, then $p = 4(g + 1)$.

Page 3

Table 2

Min: 1, 2, 3, 4, 5, 6, . . .

Max: 1, 2, 4, 6, 9, 12, 16, 20, 25, . . .

6. 14, 17, 21
7. Min. area: Subtract 2 from the perimeter and divide the result by 2 $A = (p - 2)/2$
8. 56, 81, 110
9. Max area: Let q = the greatest integer in $P/4$ (whole-number quotient of $P/4$). If P is a multiple of 4, then the maximum area is q; otherwise the maximum area is $q(q + 1)$.

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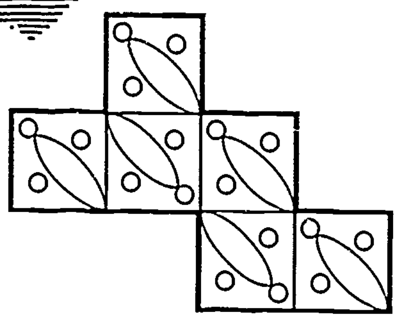
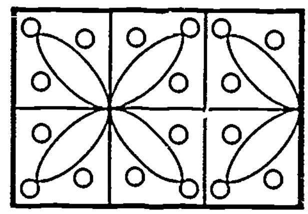
Investigating Perimeter and Area

DeCora and DeCore, interior decorators, are constructing mosaic designs using colored ceramic square tiles. Each square is the same size, and the length of a side is one unit. With six squares, they created the following designs:



Design 1



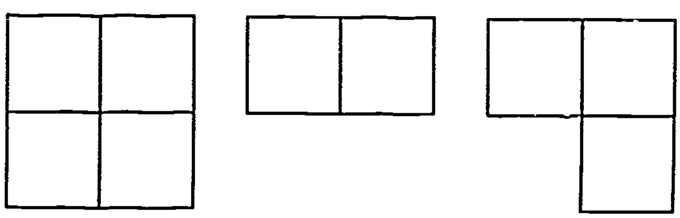
Design 2

It is interesting to note that even though the areas are the same, the perimeters are different. Find the perimeter of each figure to see how they differ. To determine the price of a mosaic, DeCora and DeCore charge $8 for each tile and $5 per unit of length for framing (1 unit of length = the length of one side of the tile).

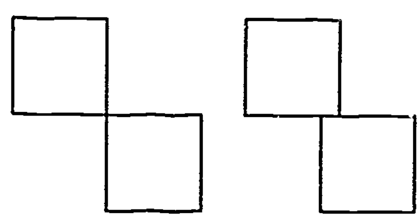
- Find the total cost of the tile and framing for each mosaic shown above.

Design 1 = ____________ Design 2 = ____________

DeCora and DeCore often have to explain to customers that designs with the same area may have different total costs, so they need a good understanding of the relationship between the perimeter and area of polygonal figures. The following activities offer an opportunity to explore the relationship between perimeter and area using arrangements of squares. In each arrangement, neighboring squares must share a common side.



(These arrangements can be used.)



(These cannot be used.)

Investigating Perimeter

You will need grid paper and fifty to one hundred square tiles or squares cut from construction paper. Let the length of the side of a square be one unit. Note that the area of each arrangement is equal to the number of squares. Work with a group of classmates to investigate the perimeter of certain arrangements found by putting squares together.

The editors wish to thank Judy Mumme, mathematics department of the University of California at Santa Barbara, for writing this issue of *NCTM Student Math Notes.*

Investigating Perimeter—Continued

- Verify the results in table 1 for one, two, and three squares.
- Explore different arrangements of squares to determine the perimeter for each.
- Explore the range of perimeters possible for each number of squares to find the arrangements that produce the maximum and minimum perimeters.
- Use grid paper to sketch the arrangements for the maximum and minimum perimeters.
- Record the maximum and minimum perimeters for the area arrangements in table 1.
- Look for patterns in the table and on the grid paper to see if you can predict the maximum and minimum perimeter of an arrangement. You should extend table 1.

Table 1

Area (No. of Sq.)	Maximum Perimeter	Minimum Perimeter
1	4	4
2	6	6
3	8	8
4	10	
5		10
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		

1. Can you find an arrangement for which the perimeter is an odd number? If so, draw the arrangement. If not, explain. ______

2. Predict the *maximum* perimeter of arrangements of the following numbers of squares:

   40 ______ 75 ______ 100 ______

3. Make a conjecture for finding the *maximum* perimeter of arrangements of any number of squares.

4. Now, try to predict the *minimum* perimeter of arrangements of the following numbers of squares:

   36 ______ 48 ______ 55 ______

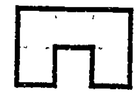
5. Make a conjecture for ..ding the *minimum* perimeter of arrangements of any number of squares.

Investigating Areas

Using the same rule for arranging squares, explore the minimum and maximum *area* of an arrangement of squares with a *given perimeter*.

- The arrangement shown in the grid has a perimeter of 10. Outline five other arrangements that have a perimeter of 10. Indicate the area of each arrangement on the grid.

- The arrangement shown on this grid has a perimeter of 12. Outline as many other arrangements as you can that have a perimeter of 12. Indicate the area of each.



- Continue to explore different arrangements of squares that produce a given perimeter and determine the area of each arrangement in order to find arrangements that produce the *minimum* and *maximum* areas. Record these values in table 2.

Table 2

Perimeter	Minimum Area	Maximum Area
4	1	1
6	2	2
8	3	4
10	4	
12		9
14		
16		
18		
20		
22		
24		
26		

6. Predict the *minimum* area for the following perimeters:

    30 ____________ 36 ____________ 1 ____________

7. Make a conjecture for finding the *minimum* area.

____

____

____

8. Predict the *maximum* area for the following perimeters:

    30 ____________ 36 ____________ 42 ____________

9. Make a conjecture for finding the *maximum* area.

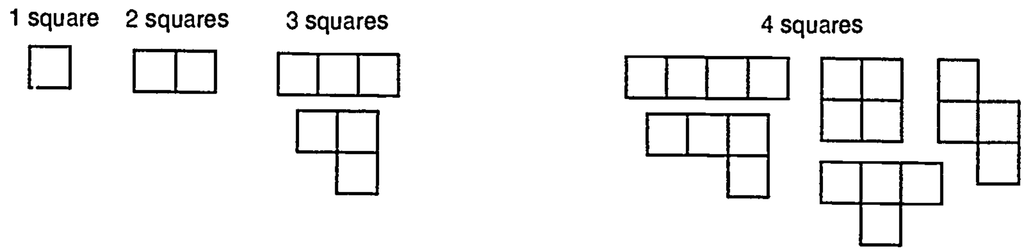
____

____

____

Did You Know That . . .

- using the same rule as before, only one way exists to arrange one square, one way to arrange two squares, two ways to arrange three squares, and five ways to arrange four squares?



- you can add squares to an arrangement to increase the area and decrease the perimeter?
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